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Vertical excitation of axisymmetric liquid bridges

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ABSTRACT

This study presents an analytical model of the dynamics of an axisymmetric liquid bridge confined between two circular pads and subjected to small vertical periodic perturbations. Such system finds important applications in microassembly and microjoint design, where force and damping need to be precisely controlled. The liquid bridge is modelled by an equivalent spring/dashpot/mass system characterised by the spring constant k, the damping coefficient b and the equivalent mass m, respectively. An abacus for k as well as analytical approximations for k, b and m based on simplifications of the Navier–Stokes equation are provided. The study is validated by experiments and numerical simulations of the system. We describe the experimental setup we designed to investigate vertical forces arising on the bottom pad from small periodic perturbations of the top pad confining the liquid meniscus. The setup allowed the accurate control of all physical and geometrical parameters relevant for the experiments. The parameters we investigated are both physical (viscosity and surface tension of the fluid) and geometrical (the edge angle between the meniscus and the pad, the height of the meniscus). The good agreement between model predictions and results let us conclude that k, b and m involve only one physical property of the liquid, namely the surface tension, the viscosity and the density, respectively.

1. Introduction

Liquid bridges are liquid volumes confined between two solid surfaces and surrounded by another fluid (most commonly, air). Mechanically, liquid bridges can be considered as joints between two solids (e.g. a substrate and a component). Upon perturbation, they generate capillary forces that can be repulsive or attractive depending on several properties of the fluid and the bounding solids. These properties are both geometrical and physical [1–3].

Forces generated by liquid bridges are ubiquitous, and technologically relevant for e.g. flip-chip electronic assembly [4–6] and capillary self-assembly of micro- and nanosystems [7,8]. Analytical models and quasi-static numerical simulations – typically performed through Surface Evolver [9] – for such applications are widely reported [3,10]. Different degrees of freedom are thereby strained, and the restoring forces or torques are computed. Although fully three-dimensional, these numerical models do not contemplate dynamics.

Dynamical studies were introduced by van Veen [11] and Meurisse and Querry [12]. They proposed analytical models for dynamical parameters and restoring forces, and for the pressure of the liquid bridge as a function of the inverse of its gap, respectively. Cheneler et al. proposed an analysis of a liquid bridge to develop a micro-rheometer [13]. The liquid is modelled as a dashpot in series with a calibrated spring/mass system. By measuring the phase shift between the position of the mass and the force exerted, the friction can be deduced. However, inertial effects are not included in the model, and the study is not supported with experimental data.

Engmann et al. [14] presented a comprehensive review of main analytical expressions of viscous force corresponding to several squeeze film configurations, e.g. through slipping, non-slipping, viscoelastic, viscoplastic. In particular, the expressions of the viscous term developed by Pitois et al. [15] can thereby be recovered.

Concerning numerical simulations, Boufercha et al. [16] estimated the time response and the position error of a self-positioning process of a chip on a substrate. The simulation includes the motion of a liquid drop crushing a substrate made of hydrophobic and hydrophilic regions, and the squeezing of the drop by the chip. Lu and Bailey [17] devised a model to determine the timescale of a chip self-alignment process. Their approach consists in coupling the motion of the solder driven by the chip, and of the chip itself. The coupling is justified by the identical timescales of the motion of both elements. They concluded that the usual, uncoupled model underestimates the impact of viscosity.

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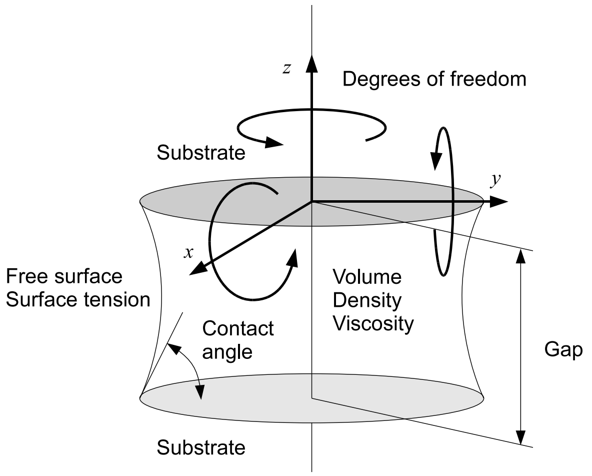

Fig. 1. Degrees of freedom for a liquid bridge between a couple of solid interfaces. The substrates refer to generic solid surfaces. They can belong to a couple made of e.g. a gripper (top) and a component (bottom), or of a component (top) and a substrate (bottom).

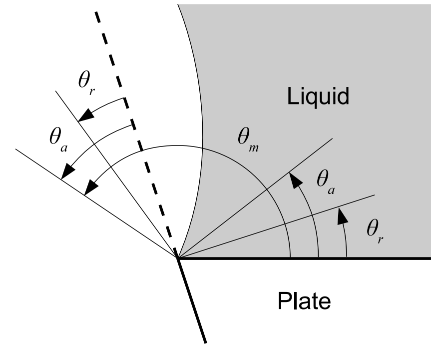

Fig. 2. Pinning of the triple contact line. When the liquid is pinned on a sharp edge, the edge angle can be higher than the advancing contact angle.

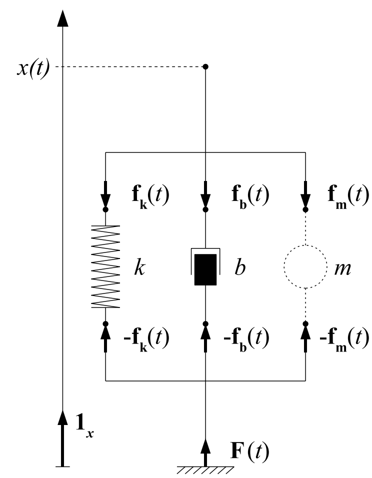

Fig. 3. The Kelvin–Voigt model. Forces $f_k(t)$ and $f_b(t)$ represent the force of the spring and of the dashpot, respectively, while $f(t)$ represents the total force exerted by liquid bridge on the system. Origin is assumed at the free length position. The direction of forces are represented for a stretched spring $x(t) > 0$, upwards velocity $\dot{x}(t) > 0$ and upwards acceleration $\ddot{x}(t) > 0$.

Other problems involving the free liquid surface were addressed in a rather different way by Montanero [18,19]. He studied the possible oscillations of an air/liquid interface. In this case, the deformation is due to the inertia of the fluid, and fluids of low viscosity are required. Beyond the theoretical consideration of inertial effect, the vibration of the interface loses interest in the problem addressed in the present work. As it will be shown, the effect of the capillary force is completely negligible at frequencies for which the shape of the free interface presents wavelets. Finally, dynamical studies of lateral forces of liquid bridges were conducted by Lambert et al. [20], including the effect of viscosity as well as free surface forces.

In this paper, we consider an axisymmetric liquid bridge periodically excited along its vertical axis. The dynamic behaviour of the liquid bridge is modelled by an equivalent spring/dashpot/mass system characterised by the spring constant k, the damping coefficient b and the mass m. The study provides an abacus and analytical approximations (by means of both a parabolic and a circular model) for k, as well as analytical laws for b and m. The analytical predictions are compared with experiments and numerical simulations. The good agreement obtained allows us to confirm the assumptions of the model, described in the following section.

2. Liquid bridge model

When the substrate (bottom solid) is fixed, the component (top solid) standing on top of the liquid bridge has six degrees of freedom with reference to the substrate: three translational and three rotational (Fig. 1). For small periodic perturbations along the degrees of freedom, the dynamic response of these degrees of freedom may be decoupled into six frequential responses. The responses can be described in several ways, depending on how the system is excited (the input) on one hand, and on how the effect of the excitation (the output) is passed on, on the other.

This study focuses on the translational degree of freedom along z (defined in Fig. 1). The liquid bridge is pinned by two circular and parallel interfaces providing an axially-symmetric geometry around the z axis. All liquid bridge dimensions are smaller than the liquid capillary length.

2.1. Edge angle

As described later, the bounding plates of our experimental setup present sharp edges to ensure the pinning of the liquid. Liquid pinning avoids the motion of the triple contact line, and affords the advantage of a less constrained edge angle. Indeed, considering a planar surface, it is known that the liquid will recede if its contact angle is smaller than the receding angle θ_r and will advance if higher than the advancing angle θ_a. The plates are described by two surfaces – one horizontal in contact with the bridge, the other nearly vertical defining the edge – on which receding and advancing angles are considered (as shown in Fig. 2 for the bottom plate). In this case, the liquid will recede if the contact angle with the horizontal surface is smaller than θ_r, and will advance only if the contact angle on the edge surface is higher than θ_a. Considering the horizontal plane as reference, the liquid will remain as long as the contact is between θ_r and θ_m Fig. 2. Moreover, liquid pinning simplifies the problem because, by avoiding the triple contact line motion, the no slip condition applies.

2.2. Model

The mechanical model of the axial degree of freedom is presented in Fig. 3: a Kelvin–Voigt system made up of a spring (of stiffness k), a damper dashpot (of damping coefficient b) and an equivalent inertial force (of mass m) connected in parallel. Such a system can be described by its frequential response and is entirely defined when the coefficients k, b and m are known.

Typical input/output pairs are the position of an interface, its velocity, its acceleration and its force. The system can be completely