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A mathematical approach to submerged horizontal buoyant jet trajectory and a criterion for jet flow patterns

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ABSTRACT

This paper focused on the trajectory and flow patterns of submerged horizontal gas jet in stagnant ambient liquid. A rigorous mathematical model for the jet trajectory was developed based on geometric analysis, mass conservation, and momentum. Additional, multiple N_2-Water experiments employing binary image processing were designed and performed. The experimental procedures and mathematical derivations are presented here in details. To assess the ability of the mathematical model to predict the jet trajectory, extensive experimental results were extracted for comparison with the mathematical results. The mathematical results were found to be in good agreement with the experimental results. Furthermore, a dimensionless jet spreading angle model was proposed and fitted to the experiment results. A flow pattern transition criterion based on the penetration depth of the buoyant jet in the water was also discussed, distinguishing the gas buoyant jet form bubbly or transitional flow patterns.

1. Introduction

The mechanics of gas-liquid plumes rising through infinite stagnant liquids have been of fundamental interest in the field of two-phase flow [1,2,10], given their wide application in industrial processes such as ladle metallurgy devices [3–6,28], and smelting furnaces [7–9]. In such cases, the gas-liquid plume is generated by a submerged gas jet, and serves several purposes including homogenizing the melt, improving the specific surface area of gas, and enhancing the rates of refining reactions. As such, it has always been characterized by a high flow rate gas jet with sufficiently mixing between gas and liquid [10,11].

With the development of the smelting industry, submerged jets with high gas flow rates and two-phase plumes have attracted increased interest from researchers, especially regarding trajectory, penetration depth, bubble distribution, velocity and other related characteristics [3–6,12–16]. However, the behaviors and properties of gas jets and plumes are complex and involve many factors, that are difficult to analyze in a quantitative, mathematical manner such as the injection gas flow rate, buoyance, viscosity, and mixing processes. Thus, numerical and empirical fitting methods [3–5,20–21,29] combined with experiments have been employed in almost all the relevant studies. Valencia and Rosales et al. [9] numerically studied and experimentally visualized the fluid dynamics in a copper converter using a VOF model, and reported that their numerical simulation could predict the gas jet formations. Buwa and Ranade et al. [17] computationally and experimentally studied the effects of gas velocity and sparger design on the bubble distribution and coalescence or break-up in a rectangular bubble column. Mudde and Simonin [18] and Delnoij and Kuipers et al. [19] performed simulation works investigating gas-liquid bubble plumes with different two-fluid models and turbulence models for comparison with published experimental findings in literatures. In the other hand, empirical fitting with experimental research has been more widely employed for practical uses [4,5]. Stapurewicz and Themelis [20] performed experiments investigating mixing times and mass transfer coefficients in an aqueous flow model with a bottom-injected gas jet and found that a low gas flowrates and porous plug injection will resulted in increased gas absorption rates in bottom-injected gas-liquid reactors. Leitch and Baines [21] measured the volume flux of a bubble plume in a homogeneous liquid and found that the volume flux was proportional to the square-root of air flow and increased linearly with height. Sano and Makino et al. [29] organized an experimental regarding the plume radius and gas holdup time of a nitrogen jet in mercury to evaluate factors such as gas flow rate, two phase densities, surface tension, and liquid depth, and produced a design formula for bath depth during reactor design. More recent progress in gas-liquid jets and plumes was made by Irons and Senguttuvan et al. [4], who focused

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Nomenclature

$a,b,c,\ k,m,n$	undetermined coefficients (–)
d_i	jet element diameter of one jet element (mm)
d_0	nozzle diameter (mm)
u_i	velocity of one jet element (m/s)
u_x	horizontal velocity of one jet element (m/s)
u_y	vertical velocity of one jet element (m/s)
u_0	velocity of gas jet at nozzle (m/s)
α_i	gas volume fraction of one jet element (–)
α_0	gas volume fraction of gas jet at nozzle (–)
Q	gas flow rate (slm)
C	criterion number for buoyant jet (–)
I	inertia force of buoyant jet element (N)
υ_l	liquid kinematic viscosity (m^2/s)
$*$	stands for dimensionless variable (–)
u_g	velocity of gas at the nozzle exit (m/s)
θ	inclination angle of buoyant jet centerline (–)
θ_c	spreading angle of jet (–)
s	motion distance of jet element (m)
V_i	volume of one jet element (m^3)
ρ_l	density of liquid (kg/m^3)
ρ_g	density of gas (kg/m^3)
M	jet momentum flow rate (N)
M_0	initial momentum flow rate of gas jet (N)
Fr'	modified Froude number (–)
Fr_p	Froude number of plume flow (–)
Re	Reynold number of liquid (–)
g	gravitational acceleration (m/s^2)
h	penetration depth of buoyant jet (m)
H^*	dimensionless penetration depth (–)
u_l	velocity of liquid around nozzle exit (m/s)

on the fundamental aspects of fluid mechanics in unconstrained gas-liquid plumes and proposed a unified analytical framework relevant to the ladle metallurgy practice. They then developed a semi-empirical and semi-theoretical method to describe the fluid dynamic and similarity characteristics. Additional, relevant works can be found in the studies of Kotsovinos and List et al. [10,22–24] who performed measurements with a plane heated water jet, and Chu and Joseph et al. [25–27] who directly measured the entrainment flow rate into a plane.

However, the works listed above primarily focused on a vertical bottom jet. Studies on horizontal gas jets, which could also be regarded as one of the most basic problems related to two-phase buoyant jets, may provide more useful values for operating condition control strategies and performance improvements in the industrial smelting processes. As such, horizontal buoyant jets and plumes trajectories should be considered a fundamentally new problem separate from vertical bottom jets. Some properties of horizontal jets, such as the spreading angle, penetration distance, and bubble distribution could significantly help to develop a deeper understanding of jet characteristics. In this manner of research, the most pervasive approach has been to conduct experiments with empirically fitting models to the data, as is represented in the works from Harby and Chiva et al. [16] and Neto and Zhu et al. [31]. Harby and Chiva regarded the jet trajectory and penetration as results of the gas Froude number (*Fr*), and then fitted their experimental data to develop empirical equations, just involving *Fr* to describe gas jets and plumes in varied conditions [16]. Neto and Zhu established a different gas-water jet trajectory empirical equation using the same method with an additional variable, the ratio of gas to water. More recent works applying a mathematical approach, can be found in Lee and Chu [27] and Themelis and Tarassoff [28] works, both of whom employed the Lagrangian approach. However, some inappropriate assumptions were introduced in both their mathematical models. As such their mathematical models may be invalid for predicting gas-liquid jet and plume trajectories in some industrial applications if these assumptions do not hold. In the work by Themelis [28], the diameter of the traced jet element was assumed to increase proportionally with the horizontal distance from nozzle, which would lead to mistakes when the buoyancy force is taken in consideration. And the work by Lee [27] was based on a gas-solid plume, and the velocity in its mixing layer was assumed to vary linearly, which may be different in gas-liquid jets and plumes.

This work aimed to explore a mathematical approach based on both less and more precise hypotheses developed in published research [28–30]. From these, a set of simultaneous equations for the buoyant gas jet trajectory would be derived. Additionally, a series of experiments would be carried out to verify the feasibility of this mathematical solution. The spreading angle and criteria for submerged gas buoyant jets would be also measured and discussed.

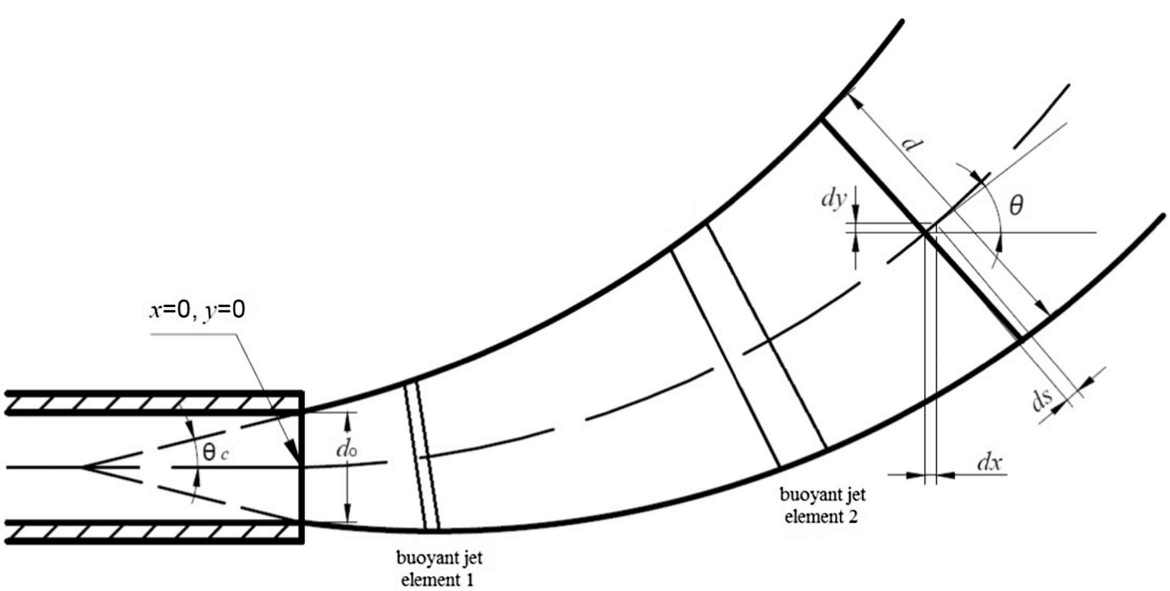

Fig. 1. Geometric schematic diagram of two-phase jet.