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Experiments on nonlinear harmonic wave generation from colliding internal wave beams

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ABSTRACT

Internal waves are abundant in both the ocean and atmosphere. Their propagation and breaking are essential to energy transfer and dissipation. However, nonlinear generation of harmonic waves due to interactions among internal waves of the same scale has not been adequately explored experimentally. When two nonresonant internal waves collide, harmonics are formed at the sum and difference of multiples of the colliding waves' frequencies, transferring energy from the initial wave beams to the harmonics. Here we experimentally explore interactions between nonresonant internal waves of the same scale and determine the relative kinetic energy transfer to their harmonics for eight unique configurations. We compare the harmonics generated here to those determined through the analysis of Tabaei et al. (2005) [1] and Jiang and Marcus (2009) [2]. It is found that approximately 7–16% of the original relative kinetic energy of the two interacting waves is transferred to the harmonics discussed here. For these configurations this value is more dependent on the relative direction the colliding waves approach each other than on their particular frequencies.

1. Introduction

Internal waves are consistently generated in continuous, stably stratified fluids, such as the ocean and atmosphere. In these media the density of the fluid increases continually with depth, due to salinity and temperature (ocean) or just temperature (atmosphere). As this stable stratification is disturbed, fluid particles are moved to regions where they are no longer neutrally buoyant, and will begin to oscillate. These motions generate internal waves. In recent decades, it has been found that internal waves have a non-negligible effect on the transfer and dissipation of energy in both the atmosphere and ocean [3]. The energy transferred by internal waves contributes to sustaining deep ocean life through ocean mixing [4] and can affect global climate patterns [5] through altering the global energy distribution. In an attempt to investigate how internal waves are generated, interact with surrounding phenomena, and dissipate aids in understanding how internal wave energy transfer affects the global energy distribution. Simplified linear models have been used extensively to estimate the generation, propagation, and dissipation of internal waves. Unfortunately, nonlinear effects can make significant contributions to energy exchange among internal waves and much is still to be learned in this area.

In an attempt to investigate the nonlinear dynamics of internal wave propagation multiple studies have focused on internal waves interacting with realistic phenomena. These include wave propagation through vortices [6,7], shear [8–12], density discontinuities [13–15], sheared density variations [16], solid boundaries [1,17–21], and other internal waves [2,22–27]. When these interactions are nonlinear, complexities are introduced and harmonic wave generation often occurs.

A particular type of wave–wave interaction which has been studied extensively is a resonant wave–wave interaction. In this situation two internal waves collide and during the interaction their energy is transferred with a third wave such that $\omega_1 \pm \omega_2 = \omega_3$ and $\mathbf{k_1} \pm \mathbf{k_2} = \mathbf{k_3}$ where ω is the wave frequency and $\mathbf{k}$ is the total wavenumber. The frequency, ω, satisfies the dispersion relation,

$$\omega^2 = N^2 \frac{k^2 + l^2}{k^2 + l^2 + m^2}, \tag{1}$$

where k and l are the horizontal (x and y, respectively) wavenumbers and m is the vertical (z). N is the Brunt–Väisälä, or buoyancy, frequency and must be constant in (1). Although N varies throughout the depth of the ocean, it is nearly constant in the deep ocean and we will assume it is constant here. N is defined by

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$$N^2 = -\frac{g}{\rho_0}\frac{\partial\bar{\rho}}{\partial z} \tag{2}$$

where $\bar{\rho}$ is the average density variation as a function of depth, g is gravity, and ρ_0 is the reference density. Internal waves only exist at frequencies less than that of the buoyancy frequency.

Resonant wave–wave interactions were first believed to be important when introduced by Phillips [28]. They have since been the focus of many studies [29–32] which have shown their significant contribution to energy transfer between frequencies in the Garrett and Munk spectrum [33]. Parametric subharmonic instabilities (PSI) follow this resonant condition, wherein a large scale (primary) wave is perturbed and, following the resonance conditions above, it begins to emit two waves of approximately half the primary wave frequency at smaller scales. Recent studies have shown this may be an effective mechanism by which internal wave energy is transferred to small enough scales that breaking may occur [34–38]. Internal wave reflection also follows the resonant condition, and Thorpe created an analytical model of nonlinear plane wave reflection at a density interface and found harmonics generated at twice the original wave frequency [14]. Higher harmonics were also found during reflection from a sloping solid surface in numerical models [1], experiments [17,21], and ocean observations [18].

Despite the prevalence of studies on resonant wave–wave interactions, there have been relatively few studies concerning nonresonant wave–wave interactions. These are wave–wave interactions which do not conform to the resonance condition, however waves of harmonic frequencies may be generated due to the nonlinear collision of the waves. Tabaei et al. [1] and Jiang and Marcus [2] estimated analytically the expected propagation directions of waves of harmonic frequencies generated during an interaction between waves of nearly the same scale. As with resonant wave–wave interactions, when two nonresonant internal waves collide, harmonics at the sum and difference of multiples of the colliding waves' frequencies are formed. This phenomena can be characterized by

$$\omega_{harmonic} = |P\omega_1 \pm Q\omega_2|, \tag{3}$$

where ω_i represents the frequencies of the colliding waves ($i = 1, 2$), $\omega_1 > \omega_2$, and P and Q represent any pair of positive integers. Lower order harmonics, where P and Q are small, are generally more important than higher order harmonics, and only harmonics with $\omega < N$ can develop into propagating internal waves. Harmonics will only form if energy is transferred from the colliding waves to the harmonics. To the author's knowledge, no previously performed laboratory experiments have attempted to quantify the transfer of energy from two colliding internal waves of the same scale to generated harmonics.

A nonresonant wave–wave interaction was created by McEwan [39] as he explored the impact of interactions on the continuous stratification of the fluid, but there was little focus on generated harmonics. Chashechkin and Neklyudov [40] found harmonic frequencies present in their experiments by inserting conductivity probes in and around the interaction region of two colliding waves. They found the amplitudes of the generated harmonics but did not quantify the energy transferred to the harmonics. Internal wave interactions were visualized by Teoh et al. [41], but no harmonic internal waves were reported in this case due to symmetry and the harmonic frequencies being higher than the Brunt–Väisälä frequency. Instead, energy accumulated in the evanescent harmonics until the fluid eventually overturned. Javam et al. [42] performed numerical studies on interacting internal waves and confirmed that if harmonic energy could not leave the interaction region in the form of propagating waves, overturning would ensue. On the other hand, if propagating harmonic waves were formed, the harmonics would have frequencies in accordance with (3), and the stratification would not be destroyed. Numerical studies were also performed by Huang et al. [43] in their study of nonresonant interactions in the atmosphere. The analytical work of Tabaei et al. [1] derives equations predicting the amplitudes of harmonics generated by two colliding internal wave beams assuming weakly nonlinear theory. Their derivations predict that up to six first-order harmonic waves are generated, all at different amplitudes. These selection rules were augmented by Jiang and Marcus [2], who created a complete set of directional propagation rules for the first-order harmonics generated during these interactions.

This study performs laboratory experiments to visualize the two-dimensional flow field when two nonresonant internal waves collide. We compare qualitative results to Tabaei et al. [1] and Jiang and Marcus [2], and determine the total kinetic energy transferred to harmonics. As the two waves interact, harmonics are generated within the interaction region and propagate from the interaction site at a new frequency. In particular, the first-order harmonics, where P and Q in (3) are equal to one, are analyzed as well as another harmonic predicted by Tabaei et al. [1] with $P = 2$ and $Q = 1$. Frequencies of the colliding wave beams are chosen to ensure that these harmonic frequencies are not evanescent.

The laboratory setup and analysis techniques are described in Section 2. Results are presented in Section 3, and Section 4 contains conclusions.

2. Methods

2.1. Experimental setup

Experiments are performed in an acrylic tank of width 11.4 cm and length 250 cm which was filled to a depth of approximately 100 cm. It is filled with linearly stratified salt water using the "double bucket" method [44]. The density profile is determined by taking fluid samples at various depths. The density of each fluid sample is measured using an Anton Paar 4100 density meter which is accurate up to 0.1 kg/m^3. The buoyancy frequency is found directly from the density profile and has a typical value of $N = 1.180 \pm 0.005$ s^{-1}.

Two internal waves are created using wave generators based on the design of Gostiaux et al. [45]. Each wave generator consists of nine plates manufactured from 0.635 cm thick acrylic which form a single wavelength (Fig. 1). The plates are separated by 0.1 cm resulting in a total generator height of 6.5 cm and thus a vertical wavenumber for all generated waves of $m = 2\pi/\lambda_z = 97$ m^{-1}. The plates are 11.2 cm wide, only 0.2 cm less than the width of the tank, to ensure the generated wave is two-dimensional. Traversing through the center of the plates is a cam which is driven by a shaft

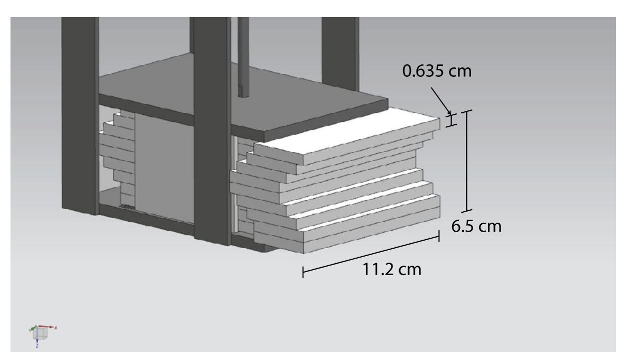

Fig. 1. Wave generator consisting of 9 acrylic plates. A rotating shaft extends into a cam through the center of the plates. The cam causes the plates to move in a sinusoid profile, generating an internal wave beam.