International Communications in Heat and Mass Transfer 97 (2018) 143–150

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Estimation for inner surface geometry of a two-layer-wall furnace with inner wall made of functionally graded materials

Wen-Lih Chen[a], Yu-Ching Yang[b,*], Haw-Long Lee[b], Win-Jin Chang[b]

[a] Department of Aeronautics and Astronautics, National Cheng Kung University, No. 1, University Rd., Tainan 701, Taiwan
[b] Clean Energy Center, Department of Mechanical Engineering, Kun Shan University, No. 195, Kun-Da Rd., Yung-Kang Dist., Tainan 710-03, Taiwan

ARTICLE INFO

Keywords:
Inverse problem
Shape identification
Furnace
Functionally graded materials

ABSTRACT

In this study, an inverse algorithm based on the conjugate gradient method and the discrepancy principle is applied to solve the inverse heat conduction problem in estimating the unknown inner wall surface geometry of a two-layer-wall furnace whose inner wall is made of functionally graded materials (FGMs). The unknown geometry is estimated from the temperature measurements taken within the furnace wall. The inverse solutions will be justified based on the numerical experiments in which two different geometry of inner surface are to be determined. The temperature data obtained from the direct problem are used to simulate the temperature measurements. The influence of measurement errors and measurement locations on the precision of the estimated results is also investigated. Results show that the unknown geometry of the inner wall surface can be predicted precisely by using the present approach for the test cases considered in this study.

1. Introduction

The inside of furnace experiences extremely severe and complex conditions after a long operating period in industry. Both corrosion effects and thermal stress can damage the furnace walls. Hence, monitoring the shape and temperature of inner surface is quite essential to ensure the safe operation of the furnaces. Chen and Su [1] and Su et al. [2] studied the shape identification problems of inner surface in furnace. In their works, they suppose that the temperature of the inner wall was known and its distribution was uniform. Wang et al. [3] solved the temperature identification problems of furnace inner surface by a fuzzy inference method (FIM). Comparisons with the conjugate gradient method and the genetic algorithm (GA) are also conducted.

On the other hand, functionally graded materials, originally proposed by Japanese researchers [4], are nonhomogeneous materials within which physical properties vary continuously to reduce the local stress concentration induced by abrupt changes in material properties. These novel materials have excellent thermo-mechanical properties to withstand high temperature and have been extensively applied to important structures, such as nuclear reactors, pressure vessels and pipes, and chemical plants [5–7]. For example, a thin functionally graded thermal shield can sustain steep temperature gradients without excessive thermal stresses. Similar advantages can be realized with functionally graded heat exchanger pipes and heat engine components, in which FGMs that continuously transit from ceramic to metallic materials would avoid the mismatch of the thermal expansion coefficient found in laminated materials.

In the past several decades, inverse analysis has been widely applied to solve engineering problems in which measurement is difficult, instruments for measurement are expensive, or the operation process is too complicated to measure directly the physical characteristics. In the heat transfer area, external inverse problems include estimation of temperature, heat flux, or heat transfer coefficient [8, 9], and internal inverse problems include determination of thermo-physical properties, such as thermal conductivity and heat capacity [10, 11]. In addition, the inverse analysis has also been applied to the problems related to shape design [12–14] and shape identification [15–17].

Although a great number of reports dealing with the shape identification problems of homogeneous mediums have been available, however, to the best of the authors' knowledge, the study on the shape identification problems of FGMs is limited in the literature. The aim of the present study is to develop an inverse analysis for estimating the unknown geometry of inner wall surface in a two-layer-wall furnace with its inner wall made of functionally graded materials, from the knowledge of temperature measurements taken within the wall. The heat conduction problem that the inverse method is applied is steady state, hence only a couple of spatial temperature measurements are needed. Here, we present the conjugate gradient method (CGM)

* Corresponding author.
E-mail address: ycyang@mail.ksu.edu.tw (Y.-C. Yang).

https://doi.org/10.1016/j.icheatmasstransfer.2018.07.009

Nomenclature

Bi	Biot number hr_3/k_2
F	unknown inner wall surface radius (m)
f	dimensionless unknown inner wall surface radius
h	convection heat transfer coefficient (W m^{-2} K^{-1})
J	functional
J'	gradient of functional
k	thermal conductivity (W m^{-1} K^{-1})
P	direction of descent
r	radial coordinate (m)
r_1	inner surface radius of furnace (m)
r_2	interface radius of furnace (m)
r_3	outer surface radius of furnace (m)
r_m	radius of temperature measurement positions (m)
T_1	temperature of inner wall (K)
T_2	temperature of outer wall (K)
T_i	temperature of inner wall surface (K)
T_∞	surrounding temperature (K)

Greek symbols

Δ	small variation quality
β	step size
γ	conjugate coefficient
ε	convergence criterion
η	dimensionless radius of furnace
θ_1	dimensionless temperature of inner wall
θ_2	dimensionless temperature of outer wall
λ	variable used in the adjoint problem
σ	standard deviation
ϕ	polar angle (radian)
ϖ	random variable

Superscripts

N	iterative number

[18–21] and the discrepancy principle [22] to estimate the unknown inner wall surface geometry by using the simulated temperature measurements. The CGM derives from the perturbation principles and transforms the inverse problem to the solution of three problems, namely, the direct, the sensitivity, and the adjoint problems, which will be discussed in detail in the following sections.

2. Analysis

2.1. Direct problem

The structure of a two-dimensional furnace wall system with two-layer walls is shown in Fig. 1. The radii of the interface between inner and outer walls and the outer surface of the furnace are r_2 and r_3, respectively. The inner radius $r_1(\phi)$ of the furnace is assumed to be dependent on the polar angle ϕ after a long time of operation in industry. In addition, the inner wall of the furnace is assumed to be made of functionally graded materials in which the thermal conductivity $k_1(r)$is a function of the radius r, and the outer wall is homogeneous furnace shell with uniform thermal conductivity k_2. To establish the physical model some assumptions are made as follows:

(1) The temperature T_i at the inner surface of furnace walls $r_1(\phi)$ is constant.
(2) The outer surface of furnace walls experiences convective heat transfer.
(3) Due to the use of FGMs for the inner wall, the contact thermal resistance between the two furnace walls is absent, and the heat flux is considered to be continuous.

Then, in the steady-state condition, the governing equations and boundary conditions for the temperature fields are stated as follows [5]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[rk_1(r)\frac{\partial T_1(r,\phi)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left[k_1(r)\frac{\partial T_1(r,\phi)}{\partial \phi}\right]=0,\ r_1\le r\le r_2,\ 0\le\phi\le 2\pi \quad (1)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T_2(r,\phi)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left[\frac{\partial T_2(r,\phi)}{\partial \phi}\right]=0,\ r_2\le r\le r_3,\ 0\le\phi\le 2\pi \quad (2)$$

$$T_1(r,\phi)=T_i,\ at\ r=r_1=F(\phi)=\text{unknown} \quad (3)$$

$$-k_2\frac{\partial T_2(r,\phi)}{\partial r}=h[T_2(r,\phi)-T_\infty],\ at\ r=r_3 \quad (4)$$

$$T_1(r,\phi)=T_2(r,\phi),\ at\ r=r_2 \quad (5)$$

$$\frac{\partial T_1(r,\phi)}{\partial r}=\frac{\partial T_2(r,\phi)}{\partial r},\ at\ r=r_2 \quad (6)$$

$$T_1(r,0)=T_1(r,2\pi),\ \frac{\partial T_1(r,0)}{\partial \phi}=\frac{\partial T_1(r,2\pi)}{\partial \phi},\ r_1\le r\le r_2 \quad (7)$$

$$T_2(r,0)=T_2(r,2\pi),\ \frac{\partial T_2(r,0)}{\partial \phi}=\frac{\partial T_2(r,2\pi)}{\partial \phi},\ r_2\le r\le r_3 \quad (8)$$

where $k_1(r)$ is the thermal conductivity of inner furnace wall.

Introducing the following dimensionless variables:

$$\eta=\frac{r}{r_3},\ \theta=\frac{T-T_\infty}{T_i-T_\infty},\ f(\phi)=\frac{F(\phi)}{r_3} \quad (9)$$

Then, the governing equations and the associated boundary conditions of Eqs. (1)–(8) can be rewritten as:

$$\frac{1}{\eta}\frac{\partial}{\partial \eta}\left[\eta k_1(\eta)\frac{\partial \theta_1(\eta,\phi)}{\partial \eta}\right]+\frac{1}{\eta^2}\frac{\partial}{\partial \phi}\left[k_1(\eta)\frac{\partial \theta_1(\eta,\phi)}{\partial \phi}\right]=0,\ \eta_1\le\eta\le\eta_2,\ 0\le\phi\le 2\pi \quad (10)$$

$$\frac{1}{\eta}\frac{\partial}{\partial \eta}\left[\eta\frac{\partial \theta_2(\eta,\phi)}{\partial \eta}\right]+\frac{1}{\eta^2}\frac{\partial}{\partial \phi}\left[\frac{\partial \theta_2(\eta,\phi)}{\partial \phi}\right]=0,\ \eta_2\le\eta\le\eta_3,\ 0\le\phi\le 2\pi \quad (11)$$

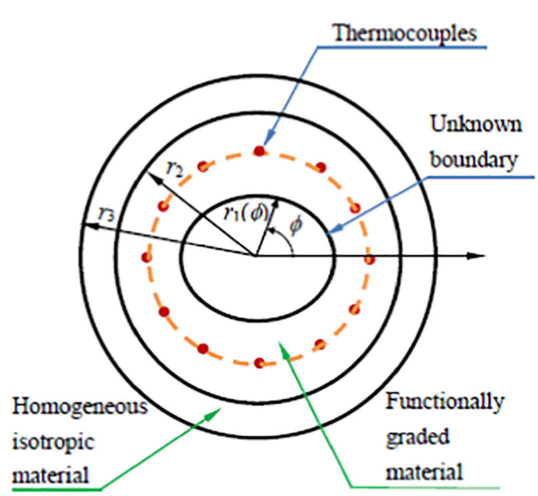

Fig. 1. Configuration of a two-layer-wall furnace.