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A homogeneous-heterogeneous model for mixed convection in gravity-driven film flow of nanofluids

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ABSTRACT

A mathematical model is constructed to analyze the mixed convection gravity-driven film flow of nanofluids in the presence of heterogeneous-homogeneous reactions. It is considered that heterogeneous reaction taking place on the surface of plate can be described by isothermal cubic autocatalator kinetics while homogeneous reaction happening at far field is given by first order kinetics. Hysteresis bifurcations incorporating the numerical method is used to obtain the multiple solutions for the system. The uniqueness of this problem lies in the fact that the multiple solutions are possible for the cases of favorable buoyancy ($\sigma < 0$), non-buoyant ($\sigma = 0$) and unfavorable buoyancy ($\sigma > 0$) for various diffusion coefficients of reactant and autocatalyst. The observed effect of nanofluids on the stability of chemical reactions in the film flow can make this problem practically useful in the related field.

1. Introduction

Thin film flow phenomenon occurs due to the motion of liquid under the influence of surface tension. This phenomenon appears in everyday life and has many applications in the field of agrochemical, nuclear reactor design, biofluid (i.e. thin films on the lungs and cornea) as well as several in industrial systems, such as, in several types of heat and mass transfer equipment like evaporators and coolers. Due to the practical significance of this phenomenon, a lot of investigation has been done by the researchers in the past with different falling film concepts. Some of the notable work in this direction was done by Andersson et al. [1]. They gave the mathematical model for flow and heat transfer in the film flow along vertical wall and presented the results numerically for Prandtl ranging from 1 to 1000. They concluded that aiding buoyancy increases the wall shear and heat transfer rate while the opposite buoyancy decreases the flow which in turns reduces the heat transfer rate. They also concluded that higher values of Prandtl number diminishes the effect of buoyancy. Andersson et al. [2] then gave the exact solution of the gravity-driven film flow using the Falkner-Skan equation for a particular value of a parameter $m = 1/2$. Then Pop et al. [3] studied the film flow along a vertical porous wall with the effects of suction and injection. They concluded that increase in the value of suction parameter increases the velocity while the decrease in injection reduces the velocity. Recently, Raees and Xu [4] investigated the convective heat transfers through the film flow along a vertical wall. They gave the explicit solutions for the problem and concluded that increasing the value of heat transfer parameter γ increases the temperature profile for both aiding and opposing buoyancy. While increase in γ increases the velocity profile for aiding buoyancy and decreases for opposing buoyancy.

Among several approaches to increase the heat transfer rate of a system, one of them is the use of nano-sized particles of different shapes and materials in the convectional heat transfer fluids to enhance their thermal conductivity. This method has been then used by many scientists to solve problems analytically and experimentally. Some of the mostly used mathematical model to describe mechanism of nanofluids includes homogeneous model [5], dispersion model [6] and Buongiorno's model [7]. Between these model, Buongiorno's model got more attention because the model considers the effect of two main slip mechanisms in the nanofluids i.e. Brownian diffusion and thermophoresis. But in the literature, only some work can be found on homogeneous and heterogeneous reactions in nanofluids. Lately, Kameswaran et al. [8] investigated the effects of homogeneous-heterogeneous reactions in nanofluids. They considered the homogeneous model for nanofluids incorporating an isothermal model developed by the Chaudhary and Merkin [9, 10] for the study of homogeneous and heterogeneous reactions. They found that the strength of the heterogeneous reaction decreases the concentration at the surface for both Ag-water and Cu-

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water nanofluids. The experimental observations on methane/ammonia and propane oxidation flowing over platinum have been given by Williams et al. [11, 12] and Song et al. [13]. Several other work that used their model to study the chemical reaction with various configurations in boundary layer flows can be found in the papers [14-20]. But all the previous work used the homogeneous model with the isothermal model and reported only one solution of the problem. The reason to consider the heterogeneous and homogeneous reactions is due to its importance in chemically reacting processes like catalysis, biochemical systems and burning, where homogeneous reaction occurs in the bulk of the fluid while heterogeneous reactions happens at catalyst surface. Recently, Zhao et al. [21] used the Buongiorno's model with the isothermal model to give the multiple solutions for the forward stagnation point flow.

Since it is known that heat and mass transfer phenomenon is greatly affected by the chemical reactions occurring in combined, forced and free convection [22], so it very much desirable to investigate the heterogeneous-homogeneous reactions with different flow regime. In literature very less attention is given in this direction. For this purpose, we analyzed the heterogeneous-homogeneous reactions in the gravity-driven film flow of nanofluid with the mixed convection. The Buongiorno's model [7] and isothermal model [9, 10] with the cubic autocatalysis is used in the present study. Also this constructed mathematical model has not been used before in literature. The transformed ordinary equations are then solved numerically and multiple solutions are obtained by hysteresis bifurcations for the non-buoyant, favorable and unfavorable buoyancy. So, the current problem is unique and has not been investigated.

2. Mathematical formulation

Consider a two-dimensional laminar thin liquid film flow containing nanoparticles falling downwards along a smooth vertical surface in the presence of homogeneous-heterogeneous reactions as shown in Fig. 1. The temperature of the fluid at the surface is kept constant T_w while at far field it is given as T_0. According to Chaudhary and Merkin [9, 10] a simple homogeneous-heterogeneous model can be used when the reaction in the bulk is isothermal cubic autocatalytic i.e.

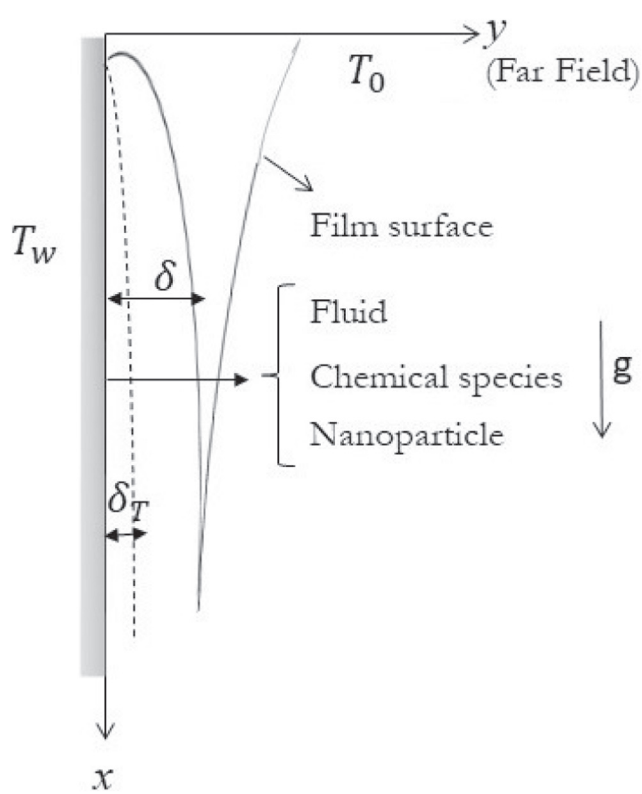

Fig. 1. Physical model for the gravity-driven film.

$$A + 2B \to 3B, \quad rate = k_1 ab^2 \tag{1}$$

while the isothermal reaction is single and first-ordered on the surface of catalyst, given by

$$A \to B, \quad rate = k_s a \tag{2}$$

whereas, a and b are the concentrations of the chemical species A and B. k_1 and k_s are the rate constants associated with homogeneous and heterogeneous reactions. We assumed that there is only one reactant A in the far field with concentration a_∞ while reactant B doesn't exists in the external flow. Further, we have considered that the heat release from the chemical reactions is negligibly small, so we do not account for the related heat transfer term in the energy equation. Following all the assumptions, the governing equations for the boundary layers, which develop along the surface, are defined as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$\rho_{f0}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \rho_f U\frac{\partial U}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} \tag{3}$$

$$+ ((1 - C)\rho_{f\infty}\beta(T - T_\infty) - (\rho_p - \rho_f)(C - C_\infty))g, \tag{4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{5}$$

$$u\frac{\partial a}{\partial x} + v\frac{\partial a}{\partial y} = D_A\left(\frac{\partial^2 a}{\partial x^2} + \frac{\partial^2 a}{\partial y^2}\right) + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - k_1\, ab^2, \tag{6}$$

$$u\frac{\partial b}{\partial x} + v\frac{\partial b}{\partial y} = D_B\left(\frac{\partial^2 b}{\partial x^2} + \frac{\partial^2 b}{\partial y^2}\right) + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + k_1\, a\, b^2, \tag{7}$$

The associated boundary conditions are

$$v = 0, \quad u = 0, \quad T = T_w, \quad D_A\frac{\partial a}{\partial y} = -D_B\frac{\partial b}{\partial y} = k_s\, a, \quad at \quad y = 0,$$
$$u = U(x), \quad T = T_0, \quad a = a_\infty, \quad b = 0, \quad at \quad y \to \infty. \tag{8}$$

where u and v are velocity components in x- and y-axes respectively, g is the acceleration due to gravity, μ is the viscosity, T is the fluid temperature, α is the thermal diffusivity, ρ is the nanofluid density, C_p is the heat capacity, D_A and D_B are the respective diffusion coefficients of species A and B, D_T is the thermal diffusion coefficient and $U(x)$ is the free stream velocity. In accordance with the boundary layer approximations, the internal dissipation of energy has been neglected and fluid properties are kept constant except the fluid density which varies with temperature. Also the buoyancy caused by the concentration of nanoparticles is negligible, so C and C_∞ has not much difference. Here, we have used the Oberbeck-Boussinesq approximation to the buoyancy terms and selected a suitable reference pressure given as

$$\frac{dp}{dx} = \rho_f U\frac{dU}{dx}, \quad U(x) = \sqrt{2gx} \tag{9}$$

Eqs. (3)–(8) can be rewritten in terms of new dependent variables, by using the following similarity transformations:

$$\psi(x, y) = f(\eta)(4U\nu x/3)^{1/2}, \quad \eta = y(3U/4\nu x)^{1/2}, \quad \theta(\eta) = \frac{T - T_0}{T_w - T_0},$$
$$\phi = \frac{a}{a_\infty}, \quad \chi = \frac{b}{a_\infty}, \tag{10}$$

This leads into the resulting set of governing equations for the momentum, energy and chemical reaction as follows: