International Communications in Heat and Mass Transfer 95 (2018) 74–79

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

A turbulent heat flux prediction framework based on tensor representation theory and machine learning

C. Sotgiu, B. Weigand*, K. Semmler

Institute of Aerospace Thermodynamics, University of Stuttgart, Pfaffenwaldring 31, 70569 Stuttgart, Germany

ABSTRACT

The need of more sophisticated cooling schemes for gas turbine blades is continuously increasing, since the hot gas maximum temperature in gas turbines has a direct influence on the engine thermal efficiency. Here, predictions of the heat transfer by means of Computational Fluid Dynamics (CFD) can complement or even reduce the number of experimental investigations. However, the modelling of the turbulent heat fluxes in the Reynolds-averaged Navier-Stokes equations heavily relies on empirical approaches. We propose a new framework for the prediction of the turbulent heat fluxes based on machine learning and tensor representation theory. A data-driven model is constructed based on the tensor description of Younis et al. (2005) and implemented in OpenFOAM. Its validation for Poiseuille flow at different Reynolds numbers shows very good agreement with reference data.

1. Introduction

The thermal efficiency of a gas turbine is strongly affected by the maximum cycle temperature reached by the hot gas, which nowadays exceeds by far the melting point of the turbine blade material [1]. Thus, sophisticated cooling systems are required to prevent engine failure, and the need for accurate heat transfer predictions by means of CFD is strongly increasing. Unsteady simulations based on Large Eddy Simulation (LES) are often prohibitively expensive at large Reynolds numbers [2,3]. Indeed, high spatial and temporal resolutions are required, and data must be sampled for adequate time periods to compute flow statistics. On the other hand, Reynolds-Averaged Navier-Stokes (RANS) methods have significantly lower resolution requirements and directly provide mean flow quantities. The averaging process results in unknown terms, the Reynolds stress tensor $-\rho\,\overline{u_i u_j}$ and the scalar flux vector $-\rho c_p\,\overline{u_i t}$. Here, u_i and t are respectively the turbulent fluctuations of velocity and temperature. Due to the complexity of the physics involved, their modelling is very challenging and mostly based on empiricism [3]. Advancements have been made in the development of models for the Reynolds stress tensor since the first two-equation turbulence model of [4]. Closures based on [5] are able to represent turbulence anisotropy with a nonlinear eddy-viscosity approach. Concepts such as the elliptic relaxation [6] and the elliptic blending [7] account for the influence of solid walls on the Reynolds stresses. Turbulence-structure tensors [8] include nonlocal information in single point-closures. Efforts have been made to improve turbulence models using machine learning algorithms [9–11]. On the other hand, the scalar flux modelling in complex flows with heat transfer is mostly based on the gradient-diffusion hypothesis [12–14]. This model represents the scalar flux vector with a Fourier-like equation

$$-\,\overline{u_i t} = \Gamma_t \frac{\partial T}{\partial x_i} \tag{1}$$

The eddy diffusivity Γ_t is usually evaluated by scaling the eddy viscosity, υ_t with a turbulent Prandtl number σ_t

$$\Gamma_t = \frac{\upsilon_t}{\sigma_t} \tag{2}$$

Since the eddy diffusivity Γ_t is isotropic, the scalar flux and temperature gradient vectors are parallel, which is generally not the case in reality [13]. Anisotropic models for the scalar fluxes have seen increasing attention in the scientific community in the last 30 years [14], but viable solutions for wall heat transfer problems of engineering interest are still limited, especially if only explicit models are considered [12,13]. A convenient approach to model the anisotropic thermal diffusivity is the use of tensor representation theory to derive an explicit algebraic formula [15]. However, the model coefficients are functions of flow field invariants whose exact form is unknown. In this paper, we propose a method for deriving functional forms for the model

Abbreviations: DNS, Direct Numerical Simulation; EBRSM, Elliptic Blending Reynolds Stress Model; LES, Large Eddy Simulation; RANS, Reynolds-Averaged Navier-Stokes; ReLU, Rectified Linear Unit; YSC, Younis-Speziale-Clark scalar flux model

* Corresponding author.
E-mail address: bw@itlr.uni-stuttgart.de (B. Weigand).

https://doi.org/10.1016/j.icheatmasstransfer.2018.04.005

Nomenclature

b	neuron bias, [−]
C_i	model coefficient, [−]
c_p	specific heat at constant pressure, J K^{-1}
f	objective function, [−]
k	turbulent kinetic energy, m^2 s^{-2}
$\dot{Q_w}$	wall heat flux, W
R_{ij}	non-dimensional Reynolds stress tensor $\overline{u_i u_j}/k^2$, [−]
$\{\mathbf{R}\}$	non-dimensional Reynolds stress tensor invariant $\sqrt{R_{ij}R_{ji}}$, [−]
Re	Reynolds number, [−]
S_{ij}	non-dimensional shear rate tensor $\frac{1}{2}\frac{k}{\epsilon}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)$, [−]
$\{\mathbf{S}\}$	non-dimensional shear stress tensor invariant $\sqrt{S_{ij}S_{ji}}$, [−]
T	mean temperature, K
T^+	non-dimensional temperature T/T_τ, [−]
T_τ	friction temperature $\dot{Q_w}/(\rho c_p u_\tau)$, K
U_i	mean velocity vector, m s^{-1}
U	mean velocity magnitude, m s^{-1}
U_τ	friction velocity $\sqrt{\lvert\tau_w\rvert/\rho}$, m s^{-1}
u_i	velocity fluctuation vector, m s^{-1}
$\overline{u_i t}$	scalar flux vector, m K s^{-1}
$\overline{u_i t}^+$	non-dimensional scalar flux vector $\overline{u_i u_j}/(U_\tau T_\tau)$, [−]
$\overline{u_i u_j}$	Reynolds stress tensor, m^2 s^{-2}
$\overline{u_i u_j}^+$	non-dimensional Reynolds stress tensor $\overline{u_i u_j}/U_\tau^2$, [−]
$\mathbf{W}$	weight matrix, [−]
$\mathbf{X}$	neural network input, [−]
$\mathbf{x}$	neuron input, [−]
x_i	coordinate, m
$\mathbf{Y}$	neural network output, [−]
$\mathbf{y}$	neuron output, [−]

Greek symbols

α	thermal diffusivity, m^2 s^{-1}
Γ_t	eddy diffusivity, m^2 s^{-1}
δ	channel half-height, m
ε	turbulent dissipation, m^3 s^{-2}
υ	kinematic viscosity, m^2 s^{-1}
υ_t	turbulent viscosity, m^2 s^{-1}
ρ	fluid density, kg m^{-3}
σ	neuron activation function, [−]
σ_t	turbulent Prandtl number, [−]
τ_w	wall shear stress $\rho\ \upsilon(\partial U/\partial x_1)$, kg m s^{-2}

coefficients using machine learning algorithms.

2. Test case

The test case is the Poiseuille flow with heat transfer, which is described in detail in [16]. Referring to Fig. 1, the Reynolds number is defined as

$$\mathrm{Re} = \frac{U\delta}{\upsilon} \tag{3}$$

where U is the mean axial flow velocity magnitude, δ is the channel half-height and υ the kinematic viscosity. The wall distance will be expressed in viscous lengths

$$y^+ = \frac{U_\tau x_2}{\upsilon} \tag{4}$$

where $U_\tau = \sqrt{\tau_w/\rho}$ is the friction velocity, $\tau_w = \rho\ \upsilon(\partial U/\partial x_1)$ is the wall shear stress, and ρ the density. The following non-dimensional temperature will also be used

$$T^+ = \frac{T}{T_\tau} \tag{5}$$

here, $T_\tau = \dot{Q_w}/\rho c_p u_\tau$ is the friction temperature, $\dot{Q_w}$ the wall heat flux applied at $x_2 = 0$ and c_p the specific heat at constant pressure. The Reynolds stress tensor and the scalar flux vector can also be represented in the following non-dimensional form

$$\overline{u_i u_j}^+ = \frac{\overline{u_i u_j}}{U_\tau^2}, \quad \overline{u_i t}^+ = \frac{\overline{u_i t}}{U_\tau T_\tau} \tag{6}$$

3. Computational setup

Fig. 1 provides a summary of the boundary conditions. We assume in the following a fully developed flow in a plane channel. Computational grids with 21, 42, 70, and 100 cells along the wall normal direction are used respectively for the Reynolds numbers 5665, 14,100, 24,428 and 41,400. The flow is assumed incompressible and the temperature is treated as a passive scalar. Source terms in the momentum and energy conservation equations allow periodicity of the flow and thermal fields [17]. The SIMPLE algorithm of [18] is used as flow solver, and all the convection terms are discretised using second-order differentiation schemes. A treatment analogous to that of the pressure in collocated grid arrangements is applied to the Reynolds stresses to improve convergence [19]. Grid independency is assessed by repeating the simulations with doubled grid resolution, where the maximum variations in U_τ and T_τ are found to be respectively within 1% and 1.5%.

4. Description of the current model

We introduce a framework for computing the scalar fluxes using tensor representation theory and machine learning. The framework is named "Italo". Functional forms for the model coefficients in agreement with tensor representation theory are derived using a big-data approach. Individual neural networks reproduce the behavior of each model coefficient, whose distribution over the computational domain is determined prior training.

4.1. Neural networks

Neural networks are arrangements of perceptrons [20], which approximate the behavior of biological neurons. A perceptron (or artificial neuron, Fig. 2a) performs the following operation

$$\mathbf{y} = \sigma(\mathbf{W}^{\mathrm{T}}\mathbf{x} + b) \tag{7}$$

where $\mathbf{x}$ is the neuron input, $\mathbf{W}$ the weights, b the bias, σ an activation function and $\mathbf{y}$ the neuron output. Both the weights and biases are training parameters, which can be tuned to approximate a data set. We use the multilayer feed-forward neural network (Fig. 2b) as universal

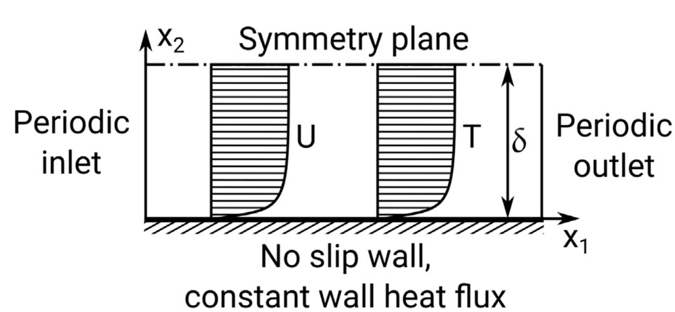

Fig. 1. Test case: Fully developed flow in a parallel plane channel.