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Phase-domain power flows in the rectangular co-ordinates frame of reference including VSC-based FACTS controllers

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Abstract

This paper presents a method for systematic modelling of VSC-based FACTS controllers within three-phase power flows in rectangular co-ordinates. Among the FACTS controllers modelled are the STATCOM, SSSC, UPFC and HVDC-VSC. The approach taken is to represent the fundamental frequency operation of each power converter as a three-phase voltage source behind a leakage reactance, where one or two of them may be connected in either series or parallel depending on the FACTS controller being modelled. Active and reactive power flow equations are developed for each voltage source circuit together with constraint equations to account for co-ordinated operation of two converters, such as in modelling of UPFC and HVDC-VSC. The power flow equations representing the VSC-based FACTS controllers are combined with the nodal power equations of the power network for a combined iterative solution using a Newton–Raphson three-phase power flow algorithm in rectangular co-ordinates enabling robust and efficient solutions of three-phase power networks with any number and kind of VSC-based controllers.

Keywords: FACTS; STATCOM; SSSC; UPFC; HVDC-VSC; Three-phase Newton–Raphson; Power flows; Rectangular co-ordinates

1. Introduction

The basic formulation of the three-phase power flow problem is already well established. Arguably, nodal analysis is the preferred frame-of-reference in this application area owing to the highly sparse nature of the nodal admittance matrix; a most desirable property which the Jacobian inherits when the power flow problem is solved using the Newton–Raphson method. The polar co-ordinates version of the Newton–Raphson method has been used more widely but the version in rectangular co-ordinates is gaining adepts on grounds of being marginally faster. The rectangular version uses the real and imaginary parts of the nodal complex voltages as state variables rather than the magnitudes and phase angles used by the polar version where the calculated active and reactive powers are expressed as a function of voltage magnitudes and phase angles [1].

Likewise, in rectangular co-ordinates, the calculated active and reactive powers are expressed as a function of real and imaginary voltage components. Despite the fact that a larger number of equations and variables exist in the rectangular formulation (by the number of PV-type buses), when sparsity programming is used, such an increase in size is of little significance. Indeed, one iteration of the rectangular method is marginally faster than one iteration of the polar co-ordinates method, because in the former there are practically no trigonometric operations to carry out [2,3].

An in-depth literature search indicates that so far, in spite of the growing importance of the subject, only little work has been reported on the representation of FACTS controllers in three-phase power flow algorithms. In particular, a comprehensive and systematic treatment of the new kind of FACTS controllers, those that use the voltage source converters (VSC) as their basic building block, has not yet appeared in the open literature. Hence, this paper presents a method to solve three-phase power flows using the

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Newton–Raphson technique based on polar and rectangular co-ordinates with provision to include models of VSC-based FACTS controllers.

2. Power flows using rectangular co-ordinates

The power flow solution of a three-phase power system may be solved very reliably by iteration using the Newton–Raphson method in rectangular co-ordinates. This involves a linearised expression comprising a vector of mismatch active and reactive powers, a vector of incremental real and imaginary parts of nodal voltages, and a matrix of first order partial derivatives; all this involving the three-phases and all nodes of the network. It takes the following form:

$$\begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial\mathbf{P}}{\partial\mathbf{e}} & \dfrac{\partial\mathbf{P}}{\partial\mathbf{f}} \\ \dfrac{\partial\mathbf{Q}}{\partial\mathbf{e}} & \dfrac{\partial\mathbf{Q}}{\partial\mathbf{f}} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{e} \\ \Delta\mathbf{f} \end{bmatrix} \tag{1}$$

where the matrix of first order partial derivatives of nodal active and reactive powers with respect to the real and imaginary parts of the complex nodal voltages is termed Jacobian.

The nodal active and reactive power equations for bus k and phase ρ are derived from the complex three-phase current injection at bus k:

$$P_k^{\rho} = e_k^{\rho}\sum_{i=1}^{n\text{bus}}\sum_{j=a,b,c}(G_{ki}^{\rho j}e_i^j - B_{ki}^{\rho j}f_i^j) + f_k^{\rho}\sum_{i=1}^{n\text{bus}}\sum_{j=a,b,c}(G_{ki}^{\rho j}f_i^j + B_{ki}^{\rho j}e_i^j) \tag{2}$$

$$Q_k^{\rho} = f_k^{\rho}\sum_{i=1}^{n\text{bus}}\sum_{j=a,b,c}(G_{ki}^{\rho j}e_i^j - B_{ki}^{\rho j}f_i^j) - e_k^{\rho}\sum_{i=1}^{n\text{bus}}\sum_{j=a,b,c}(G_{ki}^{\rho j}f_i^j + B_{ki}^{\rho j}e_i^j) \tag{3}$$

The complex three-phase voltage at bus k is $V_k^{\rho} = e_k^{\rho} + jf_k^{\rho}$, and the admittance between buses k and i, which is a 3×3 matrix block, is $Y_{ki}^{\rho} = G_{ki}^{\rho} + jB_{ki}^{\rho}$.

The Jacobian matrix and mismatch vector are evaluated at each iterative step. The generic power mismatch equations at bus k are:

$$\Delta P_k^{\rho} = (P_k^{\rho})^{sp} - (P_k^{\rho})^{\text{cal},it} \tag{4}$$

$$\Delta Q_k^{\rho} = (Q_k^{\rho})^{sp} - (Q_k^{\rho})^{\text{cal},it} \tag{5}$$

where $(P_k^{\rho})^{sp}$ and $(Q_k^{\rho})^{sp}$ are the specified active and reactive power injections, determined by the net amount of power generation and load connected at the bus; whereas $(P_k^{\rho})^{\text{cal},it}$ and $(Q_k^{\rho})^{\text{cal},it}$ are the calculated active and reactive power injections at the bus. Nodal voltages in phase a are normally initialised at one and zero for the real part and the imaginary part, respectively. Moreover, nodal voltages for phases b, and c are initialised at $-1/2 - j\sqrt{3}/2$ and $-1/2 + j\sqrt{3}/2$, respectively.

At each iterative step the nodal voltages are updated using:

$$(e_k^{\rho})^{it+1} = (e_k^{\rho})^{it} + (\Delta e_k^{\rho})^{it} \tag{6}$$

$$(f_k^{\rho})^{it+1} = (f_k^{\rho})^{it} + (\Delta f_k^{\rho})^{it} \tag{7}$$

The process is repeated until the power mismatches are within a specified tolerance, say $1e^{-12}$; a convergence criterion which is normally met within four to five iterations.

3. Voltage source converter-based FACTS controllers

The voltage source converter is the basic building block of several power systems controllers, namely VSC-based FACTS controllers, among then are the static compensator (STATCOM), the static synchronous series compensator (SSSC), the unified power flow controller (UPFC) and the new breed of high-voltage direct current links using VSCs [4]. To illustrate this point, Fig. 1 shows the schematic diagram of a generic VSC-based controller comprising two VSCs, three interfacing transformers and five switches. Transformers T1 and T2 are shunt connected with the ac system whereas transformer T3 is series connected. At least four distinct equipment functions may be realised by suitable recombination of the switching states: (i) when switches A and B are on and switches C, D and E are off, the ac systems is compensated by converters 1 and 2, both acting as STATCOMs and boosting the voltages at buses k and m, respectively; (ii) when switches A, B and C are on and switches D and E are off, the system performs the function of a back-to-back HVDC-VSC, which enables the independent injection of reactive power at both nodes while a set amount