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Effect of orientation on forced convection heat transfer from a heated cone in Bingham plastic fluids

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ABSTRACT

Forced convection heat transfer from an isothermal heated cone (apex up and down) in Bingham plastic fluids has been studied numerically. Extensive results are presented here in terms of isotherm and Nusselt number over ranges of conditions as: Reynolds number, $1 \le Re \le 100$, Prandtl number, $10 \le Pr \le 100$, Bingham number, $0 \le Bn \le 100$ and cone angle, $20^{\circ} \le \alpha \le 150^{\circ}$. The rate of heat transfer is influenced mainly by the values of *Re*, *Pr* and *Bn* and to a lesser extent by the cone angle and orientation. The present results are consolidated in terms of the modified Reynolds (Re^*) and Prandtl numbers (Pr^*) incorporating the role of Bingham number.

1. Introduction

Most substances of multiphase nature and/or structured fluids like emulsions, foams and suspensions exhibit viscoplastic fluid behavior. This class of fluids deviates from the usual Newtonian fluids in so far that such a substance exhibits a fluid-like behavior only when the applied stress exceeds the fluid yield stress. Thus the flow domain consists of both fluid-like (yielded) and solid-like (unyielded) regions. The conduction-dominated heat transfer occurring in the latter is often the overall limiting step in such applications thereby making their heating, cooling and homogenization rather difficult. Consequently, much research effort has been directed at studying the convective momentum and heat transfer in such fluids in internal and external flows, past objects of two and three dimensional shapes, e.g., see Chhabra [1], though much attention has been accorded to the case of a sphere [2,3] or a spheroid [4]. Also, most of these studies deal with the fluid mechanical aspects and the analogous heat transfer phenomenon has been studied scantly. The dependence of the average Nusselt on the Bingham number varies from one flow regime to another. Thus in the forced convection regime Bingham number augments the overall heat transfer due to the progressive thinning of the fluid-like layer next to the heated object. On the other hand, the fluid yield stress (Bingham number) has deleterious effect on heat transfer in the free convection regime. Naturally, the velocity and temperature fields in the vicinity of a cone are influenced by the cone angle and orientation and this work is aimed at studying their influence on heat transfer in the forced convection heat transfer regime. In addition, to the fundamental significance of this geometry, it has relevance in the measurement of flow and rheological properties of soils [5,6], soft solids encountered in food and personal care product testing where the cone penetration test constitutes a standard quality control method [7,8]. Conversely, this approach can also be extended to establish the effect of temperature on the yield strength and other rheological characteristics by using a heated cone and measuring the rate of heat transfer. As noted elsewhere [9], there is a very limited information available on the drag and the Nusselt number behavior of a cone, even in Newtonian fluids [10,11]. While the drag behavior of a cone translating in Bingham plastic fluids has been explored in few studies [9,12], there are no corresponding results on heat transfer available in the literature. The present study is thus aimed at analyzing the effects of Reynolds number, ($1 \le Re \le 100$), Prandtl number ($10 \le Pr \le 100$), Bingham number, ($0 \le Bn \le 100$) and cone angle, ($20^{o} \le \alpha \le 150^{o}$) and two orientations (apex up and down) on heat transfer from an isothermal cone.

2. Problem formulation

Fig. 1 shows the two orientations of the cone (diameter D; temperature $T_s > T_\infty$) exposed to a uniform stream of a Bingham plastic medium ($U_z = U_\infty$ and at temperature T_∞). The flow is expected to be steady and axisymmetric; also the physical properties of the fluid (k, C, μ_B, τ_0, and ρ_∞) are assumed to be constant as $\Delta T = T_s - T_\infty = 5K$ is small in this study. Similarly, the small value of $Br \sim 0.002$ justifies to disregard the viscous dissipation effects in the energy equation. Thus for incompressible, steady and laminar flow, the velocity and the temperature fields in the fluid are governed by the continuity, momentum and energy equations, respectively written in their non-

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Nomenclature

Bn	Bingham number ($=\tau_0 D/\mu_B U_\infty$), dimensionless
Br	Brinkman number ($=\mu_B U_\infty{}^2/k(T_s - T_\infty)$), dimensionless
C	heat capacity of fluid, $J{\cdot}kg^{-1}{\cdot}K^{-1}$
D	diameter of cone, m
D_∞	diameter of the computational domain, m
h	heat transfer coefficient, $W{\cdot}m^{-2}{\cdot}K^{-1}$
j_H	Colburn-j_H factor for heat transfer, dimensionless
k	thermal conductivity of fluid, $W{\cdot}m^{-1}{\cdot}K^{-1}$
m	regularization parameter, dimensionless
n_s	unit vector normal to the surface of the cone, dimensionless
Nu_{avg}	average Nusselt number, dimensionless
Nu_L	Local Nusselt number, dimensionless
Pe	Peclet number ($= Re \cdot Pr$), dimensionless
Pr	Prandtl number ($=C\mu_B/k$)dimensionless
Pr^*	modified Prandtl number ($= Pr(1 + Bn)$), dimensionless
Re	Reynolds number($=\rho D U_\infty/\mu_B$), dimensionless
Re^*	modified Reynolds number ($= Re/(1 + Bn)$), dimensionless
S	surface area of cone, m^2
T	temperature of fluid, K
T_s	cone surface temperature, K
T_∞	inlet fluid temperature, K
U_∞	free stream velocity, $m{\cdot}s^{-1}$
U_r	velocity component in radial direction, dimensionless
U_z	velocity component in z-direction, dimensionless
r, z	cylindrical co-ordinates, m

Greek symbols

α	apex angle of cone, *degree*
$\dot{\gamma}$	rate of strain tensor, dimensionless
η	scalar viscosity function, dimensionless
θ	temperature, dimensionless
μ_B	plastic viscosity, $Pa{\cdot}s$
ρ_∞	density of the fluid, $kg{\cdot}m^{-3}$
τ_0	fluid yield stress, Pa

Subscripts

∞	inlet condition
s	cone surface condition

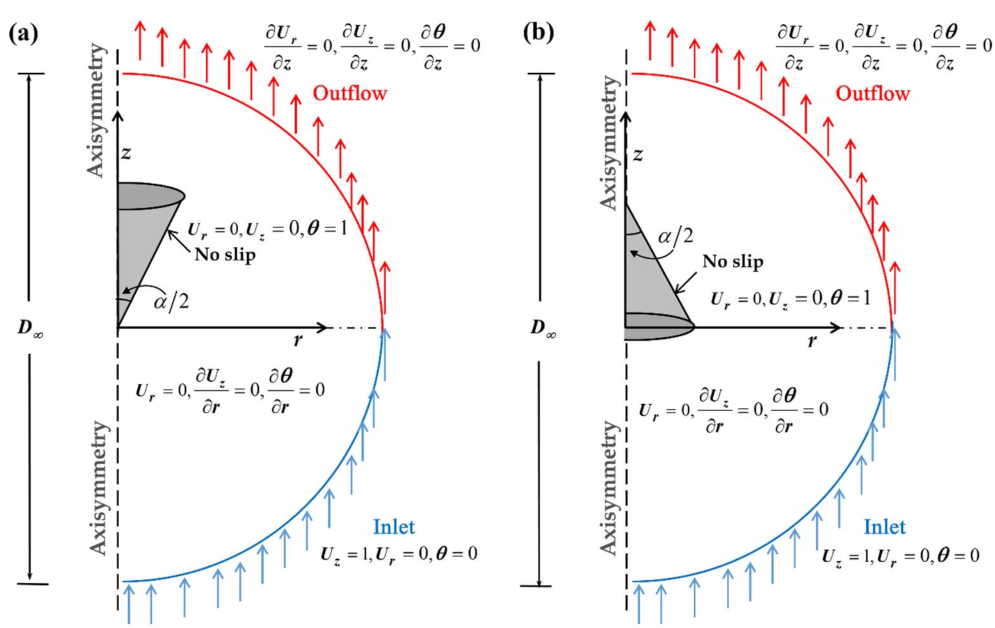

Fig. 1. Schematic of physical domains (a) configuration 1 and (b) configuration 2.

dimensional forms as: $\nabla.\ \boldsymbol{U} = 0; (\boldsymbol{U} \cdot \nabla)\,\boldsymbol{U} = -\nabla P + (\nabla \cdot \tau)/Re$; and $(\boldsymbol{U} \cdot \nabla)\,\theta = (\nabla^2 \theta)/(Re \cdot Pr)$; as detailed in our recent studies [3,4,9]. These equations have been rendered dimensionless using D and U_∞ as the linear and velocity scales. Suffice it to mention here that in the present study the modified Bingham plastic fluid viscosity ($\eta_p = 1 + Bn\,[(1 - \exp(-m\,|\,\dot{\gamma}\,|))/|\,\dot{\gamma}\,|]$) by the Papanastasiou regularization approach (with, $m = 10^5$) has been employed [13]. Admittedly, some recent studies have obviated the need for a regularization approach [14,15] but the two approaches yield quite similar results.

In this work, the unconfined flow condition is reached by surrounding the cone in a large concentric spherical envelope of fluid of large diameter D_∞, as shown schematically in Fig. 1. The boundary conditions used in the present case are that of no slip on the solid cone (isothermal), uniform flow at the inlet and the standard outflow condition at the outlet [3,4,9].

Dimensional analysis of the field equations suggests the value of the Nusselt number ($Nu_{avg} = hD/k$)to be a function of Re, Bn, Pr and α for a fixed orientation. The present numerical results are studied in terms of isotherms, local (Nu_L) and average Nusselt number (Nu_{avg}) in the ensuing sections.

3. Numerical solution procedure and choice of computational parameters

The numerical solution procedure employed here is identical to that used in our recent study [9]. The numerical mesh consists of very fine triangular cells near the surface of the cone and non-uniform quadrilateral cells in the remaining region (a typical mesh is shown in Fig. 2). The region close to the cone is meshed using free triangular cells where the minimum mesh size (δ/D) is 0.004 and growth rate parameter is 1.1 for the apex downward cone and the corresponding values for apex upward case are 0.006 and 1.1 respectively. The region away from the cone surface was meshed using non-uniform quadrilateral cells with grid expansion ratio as 1.026 for both configurations. A relative