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Effect of Bejan and Prandtl numbers on the design of tube arrangements in forced convection of shear thinning fluids: A numerical approach motivated by constructal theory

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ABSTRACT

In this work, the effects of the pressure drop (i.e. the Bejan number in dimensionless term) and of the Prandtl number have been investigated with reference to optimal geometries for maximizing the heat transfer density under forced convection of shear thinning fluids. Constructal Design associated with Design of Experiments and Response Surface methodologies have been employed to search computationally for the optimals. More specifically, after having fixed the power law index value, n, equal to 0.4, we studied the effect of the Bejan number, Be, ranging from 10^4 to 10^5 (for $Pr = 1$) and the effect of the Prandtl number, Pr, ranging from 1 to 10 (for $Be = 10^5$) on the maximum dimensionless heat transfer density. The optimal geometries here detected differ much from those referred to Newtonian fluids, as a consequence of the non-linear stress behavior with respect to strain rate. We observed that the optimal aspect ratio of the elliptical tubes, r_{opt}, highlights different (opposite) behaviours with the augmentation of Be and Pr: while r_{opt} deacreases as Be increases, it augments with higher Pr, suggesting that for flows characterized by thermal diffusivity the tubes should be more slender horizontally for better heat transfer performance. In the meantime, assigned r_{opt}, the dimensionless optimal distance between tubes, $\tilde{S}_0$, proved to be practically indepent of all the tested values of Bejan number and Prandtl number.

1. Introduction

Fluids treated in the classical theory of fluid mechanics are the ideal fluid and the Newtonian fluid. The former is completely frictioness (i.e. the shear stress is absent) while the latter has a linear relationship between shear stress and shear rate. Unfortunately, the behavior of most of the “real fluids” used in mechanical and chemical industries cannot be described by these models, since they exibit non Newtonian properties. During the Sixties Metzner [1] and Skelland [2] classified the “purely viscous non Newtonian fluids” into two categories: shear-thinning fluids and shear-thickening fluids. It is worth to mention that most of non-Newtonian fluids used in heat transfer applications exibit share-thinning behavior [3–8], so that studies of shear-thickening fluids are relatively less common in literature [9,10]. Ref. [11] illustrates the numerical study of laminar natural convection heat transfer from a heated long cylinder of square cross-section submerged in stagnant power-law fluids covering both shear-thinning and shear-thickening type fluid behaviours. Ref. [12] presents a linear stability analysis of a non-Newtonian fluid saturated porous layer. The saturating fluid is an Ostwald-de Waele type of non-Newtonian fluid. Both pseudoplastic and dilatant behaviour (which corresponds respectively to the above mentioned shear-thinning and shear-thickening) have been contemplated.

The present paper is focused on the study of shear thinning fluids by means of the numerical investigation: we relied on Constructal Design [13–15] associated with Design of Experiments and Response Surface methodologies [16] to search for the optimal configurations. The experiments were solved computationally by means of a code based on finite volume method [17].

Constructal Theory, i.e. the view of design as science, has been acurately treated in Refs [18–26]. It is based on the Constructal Law, that states “For a finite-size flow system to persist in time (to live), its configuration must evolve in such a way that provides greater and greater access to the currents that flow through it”. This line of inquiry began “accidentally” in science, with a 1997 analytical paper on the heat removal from a small electronic package by using a point-size heat sink [27]. Then, the Constructal design principle has been successfully extended practically to all the branches of science [28,29]. In the present paper we rely on Costructal Theory in its original engineering focus, i.e. the geometric optimization of shapes in heat transfer and fluid mechanics. Concerning the study of non-Newtonian fluids over

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Nomenclature

a	horizontal ellipse axis [m]
b	vertical ellipse axis [m]
Be	Bejan number
H	vertical pitch [m]
k	thermal conductivity [W m^{-1} K^{-1}]
K	consistency index [Pa s^n]
L_c	problem characteristic length [m]
L_d	downstream domain length [m]
L_u	upstream domain length [m]
m	number of verification points
n	power law index
Pe	Péclet number
Pr	Prandtl number
q'	heat transfer rate per unit length [W m^{-1}]
r	ellipse aspect ratio
S_0	distance between tubes [m]
T	temperature [K]
T_w	wall temperature [K]
T_∞	fluid temperature [K]
u	velocity [m s^{-1}]
x_i	spatial coordinate [m]

Greek symbols

α	thermal diffusivity [m^2 s^{-1}]
$\dot{\gamma}_c$	shear rate [s^{-1}]
η	apparent viscosity [kg m^{-1} s^{-1}]
η_c	characteristic viscosity [kg m^{-1} s^{-1}]
ρ	mass density [kg m^{-3}]

Subscripts

pred	predicted
obs	observed
opt	optimum

Superscripts

$(\bar{\ })$	average value
$(\tilde{\ })$	dimensionless variables

cylinders of circular and non-circular cross-sections, readers are referred to the following surveys for more information: Bharti et Al. [30], Rao et Al. [31], Chhabra [32], Tiwari et Al. [33] and He et Al. [34].

In the present paper we used a modified version of the Bejan number as dimensionless pressure drop and the scaling method in coherence with the treatment proposed by Klein et al. in Ref [35]. The aim is to explore the effects of varying the Bejan number and the Prandtl number on the optimal geometries for maximizing the heat transfer density.

2. Mathematical model

Consider the domain shown in Fig. 1. There is a row of elliptical cross section tubes with imposed pressure drop, Δp. A fluid at bulk temperature T_∞ enters the domain and flows over the tubes. The tube wall is at a prescribed temperature T_w. Symmetry allowed us to analyze the problem with reference to a representative cell, which is the region within the dashed lines in Fig. 1. In Fig. 1, S_0 is the distance between tubes, H is the vertical pitch, L_u is the upstream domain length and similarly L_d is the downstream length. We define the ellipse aspect ratio, r, as

$$r = \frac{b}{a} \tag{1}$$

with b and a given as the vertical and horizontal ellipse axis, as shown in Fig. 1. Thus, the problem characteristic length, L_c, is introduced:

$$L_c = \sqrt{ab} \tag{2}$$

so that it is proportional to the square root of the ellipse area by a factor of $\pi/4$.

According to the Constructal Principle, the flow/heat transfer architecture is not assumed in advance, but it is the consequence of allowing the structure to murph so that the best geometry is the one that has the greatest thermal exchange per unit volume. In order to do this,

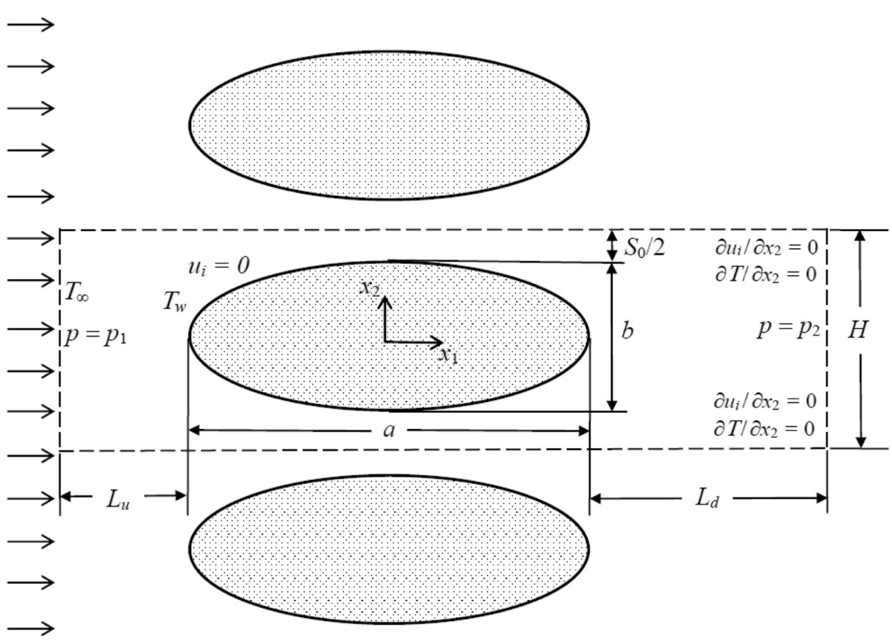

Fig. 1. Problem domain and boundary conditions.