International Communications in Heat and Mass Transfer 91 (2018) 30–39

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Description of multilayer walls by means of equivalent homogeneous models

Paola Gori[a,*], Luca Evangelisti[a,b], Claudia Guattari[a]

[a] Roma TRE University, Department of Engineering, via Vito Volterra 62, 00146 Rome, Italy
[b] Niccolò Cusano University, Department of Engineering, via Don Carlo Gnocchi 3, 00166 Rome, Italy

ARTICLE INFO

Keywords:
Heat capacity
Thermal conductivity
Multilayer wall
Heat transfer matrix
Effective properties
Homogeneous model

ABSTRACT

The accurate characterization of building envelopes represents the first step to realize a model of built structures, upon which is possible to work for the design of appropriate retrofit measures. External building walls, indeed, are one of the most significant paths of heat transfer between outer and inner environment and, as such, their control is crucial to go in the direction of energy saving. Simple models that capture the essential thermal characteristics of building walls, which are usually layered structures, would be desirable. A homogeneous wall that can be equivalent, under appropriate circumstances, to a given multilayer structure would provide an example of such a simple model. Homogeneous walls that are completely equivalent to generic multilayer walls are in general not achievable. However, we show in this work that this is possible in particular cases, i.e. if one focuses only on temperature or on heat flux behavior. It is shown that, in such situations, both an exact and an approximate solution for equivalent thermophysical properties can be devised. The evaluation of the level of approximation introduced is presented. In addition, the significance and the implications of the introduced equivalent models is discussed both regarding their exact and their approximate formulations.

1. Introduction

Buildings represent both a challenge and an opportunity to pursue the objectives of energy saving and greenhouse gas reduction: they are responsible of a large part of present-day energy requirements, but there is also a large energy-saving potential to be exploited [1,2]. One of the most effective means to achieve energy-reduction objectives is building envelope design [3]. However, such kind of design is not possible if existing buildings are dealt with and, in this case, the objective becomes their refurbishment [4]. In particular, in countries, like Italy, where historical buildings represent a significant share of the built environment, the retrofit has to be pursued by preserving their cultural integrity [5,6]. Even within such constraints, it is clear that a retrofit action can prove most cost-effective if an accurate characterization of the wall can be achieved [7]. This requires the knowledge of the thermophysical properties of the wall, such as thermal conductivity and volumetric heat capacity. Since, for existing buildings, non-destructive characterization has to be preferred, and is often mandatory, the details on the possibly heterogeneous, or simply layered, inner structure may be unknown.

The objective of determining such details may be pursued by resorting to the techniques employed for inverse problems and parameter estimation in particular [8], either with a deterministic approach [9–11] or with Bayesian analysis [12] or with stochastic search [13–15].

The first thermophysical property that needs to be determined for a multilayer wall is its overall thermal resistance, which is responsible for the steady-state behavior and can be evaluated by employing heat-flow meter measurements, whose outcomes can be processed with the moving average method [16]. Other recently proposed in-situ approaches for thermal resistance determination include: quantitative infrared thermography [17]; an excitation pulse method, based on the theory of response factors and employing two heat flow meters on the faces of the wall [18]; an iterative approach based on the Newton-Raphson method, applied to hot box measurements and to realistic experimental conditions [19].

More difficult is the evaluation of a parameter that quantifies thermal inertia effects. As a matter of fact, there is not a unique definition of such a parameter for a multilayer wall and the overall effect, in general, cannot be described by using the sum of the heat capacities of the individual layers [20,21]. This can be simply understood by observing that walls with identical composition but with different distributions of layers can behave in a largely different way, both in terms of delay and dampening of the effects produced by external stimuli [22–24]. The time constant concept has been employed as a descriptive parameter of the dynamic behavior of walls [22].

For homogeneous samples, heat capacity, together with thermal conductivity, has been determined by inverse analysis with the

* Corresponding author.
E-mail address: paola.gori@uniroma3.it (P. Gori).

https://doi.org/10.1016/j.icheatmasstransfer.2017.11.008

0735-1933/ © 2017 Elsevier Ltd. All rights reserved.

Levenberg-Marquardt minimization algorithm, with the application of a thermal stimulus on one face of the wall and measuring temperature evolution on the other face [25,26]. For heterogeneous but symmetric samples, Faye et al. [20] employed the thermal quadrupole method to build a model for determining the effective heat capacity of wall elements.

In the previous examples, points inside the wall have to be accessible to measurements, or the internal geometry of the wall has to be known, or the sample has to be essentially homogeneous. Often, however, the inner structure of the wall is not known even approximately and, to avoid destructive testing, a method that could provide an estimation of the wall thermophysical properties by using only data obtained from temperature and heat-flow measurements would be advantageous. In this context, a particularly convenient approach would be that of replacing the given structure with an equivalent homogeneous one.

The issue of the possible homogenization of the thermal properties of composite structures has been investigated in several works, for example for determining the effective properties of phase change materials [27,28], porous materials [29] or nanofluids [30]. In the cited works, the effective medium approach is applied to systems that are heterogeneous, but isotropic. In other cases, really anisotropic structures are replaced by thermally equivalent ones, with the aim of reducing the complexity of 2D or 3D structures and employ 1D models [31] or for the evaluation of the occurrence of thermal bridges [32,33].

Differently from such types of approaches, we want to analyze here the possibility of constructing effective media starting from layered building structures. It is quite intuitive that, in general, it is not possible to devise an exactly equivalent homogeneous model for layered slabs [34]. For example, asymmetric walls may not have an equivalent homogeneous representation that would be intrinsically symmetric. However, it is possible to look for an equivalent homogeneous wall that would produce the same effect as the given structure, but in specific situations only. Such an effective medium would not need to be unique for all boundary conditions, otherwise this would be in contrast with the recalled impossibility. Instead, different homogenous model walls, i.e. characterized by different effective thermophysical properties, could be determined depending on applicable boundary conditions.

In this work, we set up the theoretical framework within which the previous objective can be pursued, by showing that, within specific boundary conditions, exact equivalent analytical thermophysical properties can be determined. In addition, approximate expressions are presented for such effective properties and their applicability and limitations are discussed.

2. Theoretical modeling

2.1. Multilayer wall under steady periodic regime

If one-dimensional heat conduction can be assumed in a wall subjected to a steady periodic regime and made of a homogeneous layer characterized by a thickness L, thermal conductivity λ, and thermal diffusivity D, surface temperatures and heat fluxes can be related [34] as follows:

$$\begin{bmatrix} T_0 \\ q_0 \end{bmatrix} = \begin{bmatrix} \cosh(KL) & \sinh(KL)/Y \\ Y\sinh(KL) & \cosh(KL) \end{bmatrix} \begin{bmatrix} T_L \\ q_L \end{bmatrix} \tag{1}$$

where T_0 and T_L, q_0 and q_L are the amplitudes of the oscillating temperatures and heat fluxes at the outer and inner surfaces, respectively, and K is defined as:

$$K = (1 + j)\sqrt{\frac{\pi}{\tau D}} \tag{2}$$

In addition, $Y = \lambda K$, while τ is the time period, equal to 86,400 s for one day. T_0 can be simply a surface temperature or instead the sol-air temperature, to take solar radiation into account. Here and in the following, for the sake of brevity, we simply refer to temperature and heat fluxes to mean the amplitude of their oscillations.

Analyzing a multilayer wall, for instance a wall composed by two homogeneous layers, let us say layer A and layer B, taken in this order starting from the outside environment, and with thermophysical properties and thicknesses λ_A, D_A, L_A, and λ_B, D_B, L_B, respectively, it is possible to multiply single-layer transfer matrices of the type in Eq. (1) and obtain:

$$\begin{aligned} \begin{bmatrix} T_0 \\ q_0 \end{bmatrix} &= \begin{bmatrix} \cosh(K_A L_A) & \sinh(K_A L_A)/Y_A \\ Y_A \sinh(K_A L_A) & \cosh(K_A L_A) \end{bmatrix} \begin{bmatrix} \cosh(K_B L_B) & \sinh(K_B L_B)/Y_B \\ Y_B \sinh(K_B L_B) & \cosh(K_B L_B) \end{bmatrix} \begin{bmatrix} T_L \\ q_L \end{bmatrix} \\ &= \begin{bmatrix} C_A & \frac{S_A}{Y_A} \\ Y_A S_A & C_A \end{bmatrix} \begin{bmatrix} C_B & \frac{S_B}{Y_B} \\ Y_B S_B & C_B \end{bmatrix} \begin{bmatrix} T_L \\ q_L \end{bmatrix} \\ &= \begin{bmatrix} C_A C_B + \frac{Y_B}{Y_A} S_A S_B & \frac{C_A S_B}{Y_B} + \frac{C_B S_A}{Y_A} \\ Y_A C_B S_A + Y_B C_A S_B & C_A C_B + \frac{Y_A}{Y_B} S_A S_B \end{bmatrix} \begin{bmatrix} T_L \\ q_L \end{bmatrix} \end{aligned} \tag{3}$$

where shorthand notations for the hyperbolic functions have been employed. A homogeneous wall can be considered equivalent to the above described double layer if it gives rise exactly to the same inner temperature T_L and heat flux q_L for given outer temperature T_0 and heat flux q_0. This can happen only if the equivalent wall has the same components of the transfer matrix. For the equivalent wall:

$$\begin{bmatrix} T_0 \\ q_0 \end{bmatrix} = \begin{bmatrix} \cosh(K_{eq} L_{eq}) & \sinh(K_{eq} L_{eq})/Y_{eq} \\ Y_{eq} \sinh(K_{eq} L_{eq}) & \cosh(K_{eq} L_{eq}) \end{bmatrix} \begin{bmatrix} T_L \\ q_L \end{bmatrix} \tag{4}$$

where Y_{eq} and K_{eq} are connected to the equivalent parameters λ_{eq} and D_{eq} that have to be determined, while L_{eq} can be set equal to $L_A + L_B$, as the most simple choice. A comparison between Eq. (3) and Eq. (4) shows that, in general, it is not possible to find an equivalent homogeneous wall because the elements on the main diagonal of an arbitrary two-layer wall are different. This is the very reason why, interchanging A and B layers, the wall properties change. The main diagonal elements are equal only if the two layers are such that $Y_A = Y_B$, which means, following the definition of Y and Eq. (2):

$$\frac{\lambda_A}{\sqrt{D_A}} = \frac{\lambda_B}{\sqrt{D_B}} \Rightarrow E_A = E_B \tag{5}$$

where $E = \sqrt{\lambda \rho c}$ indicates thermal effusivity, while ρc is volumetric heat capacity. Therefore, the necessary condition for the main diagonal elements of the AB matrix to be equal is that the thermal effusivities of the two A and B materials be equal [34].

It is shown below that the equal-effusivity condition is also sufficient for the equivalence to hold. Indeed, if $E_A = E_B$, the matrix elements of the double layer in Eq. (3) can be written as follows:

$$\begin{aligned} m_{11}^{AB} &= C_A C_B + S_A S_B = \cosh(K_A L_A + K_B L_B) = \cosh(K_{eq} L_{eq}) = m_{11}^{eq} \\ m_{12}^{AB} &= \frac{C_A S_B}{Y_{eq}} + \frac{C_B S_A}{Y_{eq}} = \frac{1}{Y_{eq}} \sinh(K_A L_A + K_B L_B) = \frac{\sinh(K_{eq} L_{eq})}{Y_{eq}} = m_{12}^{eq} \\ m_{21}^{AB} &= Y_{eq}(C_A S_B + C_B S_A) = Y_{eq} \sinh(K_A L_A + K_B L_B) = Y_{eq} \sinh(K_{eq} L_{eq}) \\ &= m_{21}^{eq} \\ m_{22}^{AB} &= C_A C_B + S_A S_B = \cosh(K_A L_A + K_B L_B) = \cosh(K_{eq} L_{eq}) = m_{22}^{eq} \end{aligned} \tag{6}$$

which shows the full equivalence of the single and of the double layer.

Eq. (6) also shows that $K_A L_A + K_B L_B = K_{eq} L_{eq}$ and therefore:

Download English Version:

https://daneshyari.com/en/article/7053021

Download Persian Version:

https://daneshyari.com/article/7053021

Daneshyari.com