International Communications in Heat and Mass Transfer 91 (2018) 210–215

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

On thermal analysis of periodic composite coatings for a homogeneous conductive layer

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ARTICLE INFO

Keywords:
Composite coating
Heat transfer
Temperature
Homogenized model

ABSTRACT

The paper deals with the problem of heat conduction in a nonhomogeneous rigid body which is a homogeneous conductive layer with a periodic composite coating. The coating layer is a composite in which every unit cell in general case is composed of four rigid beams with rectangular cross-sections. The assumed structure permits to analyse some special cases of coatings: laminated composites with layer parallel or perpendicular to the boundary, composites with periodically distributed inclusions with rectangular cross-section and a homogeneous layer. The distributions of temperature and heat flux distributions on the interface between the coating and the homogenous layer are investigated.

1. Introduction

Composite coating systems are applied at hi-tech facilities to protection against corrosion, heat, fire, stress, UV light and for an assurance of waterproofing. The wide scope of practical applications resulted in many experimental and theoretical investigations of composite coatings. Within this paper periodic composite coatings on a homogenous conductive substrate are considered as regarding the heat condition. The coating layer is composed of periodically repeated four beams with rectangular cross-sections. To analyse the heat conduction in the nonhomogeneous structure, the homogenized model with microlocal parameters [1] will be applied in which the continuity conditions for temperature and heat fluxes are satisfied. The upper boundary of coating is assumed to be kept in a strip at given temperature, the remaining part of upper boundary is kept at zero temperature. The zero temperature is taken into account on the lower boundary plane of the homogeneous layer. Moreover, the continuity conditions of the temperature and the heat flux vector on the interface between the coating and the substrate are assumed. The four component structure of coating allows to analyse and compare some cases of coatings composed of insulators and heat conductors.

The aim of the paper is twofold. Firstly the applied mathematical model of heat conduction for the composite coating will be verified in a two-component periodically layered case. Secondly, the obtained solution within the framework of the homogenized model will be used to perform an analysis of the composite coating investigating different components of the composite layer.

The problems connected with the properties of material coatings are of great interest in engineering. The list of references devoted to the thermomechanical problems of the homogeneous bodies with coatings is rather long, so now only a few papers will be mentioned. A nonstationary heat conduction problem for a half-space with a multilayer coating is solved in [7], where some analytical methods are applied. The heat conduction problem connected with a rigid periodically layered half-space coated by a functionally graded layer is investigated in [8]. A deboning of graded thermal barrier coating from bases is considered in [9]. The paper [10] presents an analytical and numerical methods of solution of three-dimensional of a FGM coated half-space. Some papers are devoted to the investigations of cracks in the nonhomogeneous bodies composed of coatings and substrates. In the paper [11] the axisymmetric problem of crack in the graded layer on a homogenous half-space under thermal loadings is considered. The paper [12] presents the solution of the problem for a partially insulated interface crack between a graded orthotropic coating and a homogeneous orthotropic substrate under flux supply. Many papers are devoted to the contact problems of coated bodies. For instance, [13] solves the contact problem of a rigid punch and a homogeneous half-space coated with graded layer. In [14] the thermoelastic contact problem of a flat punch sliding over a graded coating/substrate system with frictional heat generation is investigated. The frictional sliding contact problems of rigid parabolic and cylindrical stamps on a graded coating are considered in [15]. The analysis of distribution of friction heat during cold-rolling of metals by using composite rolls is presented in [16].

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https://doi.org/10.1016/j.icheatmasstransfer.2017.12.015

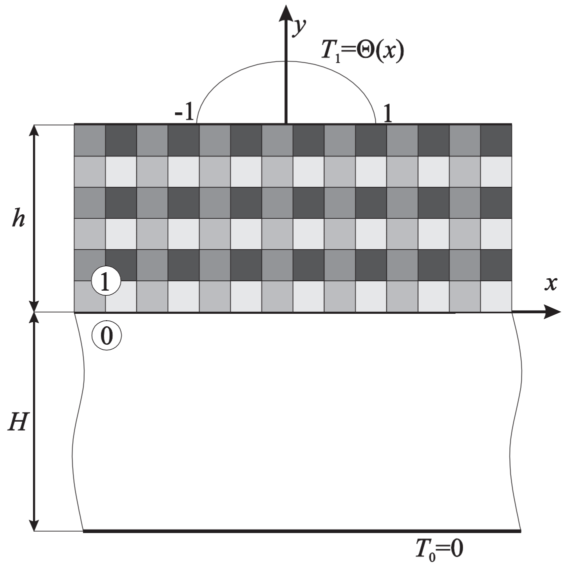

Fig. 1. Scheme of considered problem.

2. Statement and solution of the problem

Consider a rigid homogenous layer with a periodic composite coating heated on the upper surface in a strip. Let c denote the half of width of the heated region. The considered two-dimensional problem will be formulated in the dimensionless Cartesian coordinates (x,y) related to the dimension c. The nonhomogeneous body is composed of homogeneous substrate in the region $-\infty < x < \infty$, $-H \le y \le 0$, where $H^* = Hc$ is the thickness of the foundation, Fig. 1, and a periodic composite coating in the region $-\infty < x < \infty$, $0 \le y \le h$, where $h^* = hc$ is the thickness of the coating layer.

Let k_0 be the thermal conductivity coefficient of the substrate. The coating layer is assumed to be composed of four periodically repeated rigid beams with rectangular cross-sections. The cross-sections of unit cell is shown in Fig. 2. Let k_i, $i = 1,2,3,4$ be thermal conductivities of the composite components, respectively. Let Δ_i, i = 1,2,3,4, be the regions occupied by the i-th kind of material, Fig. 1. Here the regions are given as follows: $\Delta_1 = \{(x,y), 0 \le x \le a_1, 0 \le y \le b_1\}$, $\Delta_2 = \{(x,y), a_1 \le x \le a_1, 0 \le y \le b_1\}$, $\Delta_3 = \{(x,y), 0 \le x \le a_1, b_1 \le y \le b\}$, $\Delta_4 = \{(x,y), a_1 \le x \le a, b_1 \le y \le b\}$, where $a_1{}^* = ca_1$, $b_1{}^* = cb_1$, $a^* = ca$ and $b^* = cb$ are the dimensions of the composite components in the unit cell.

Let the upper boundary be kept at given temperature $\theta(x)$ for

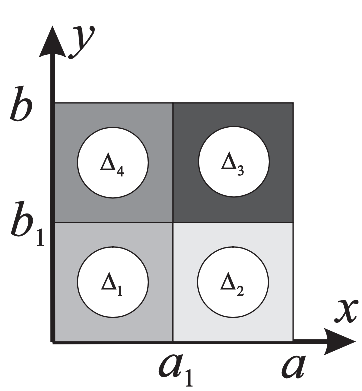

Fig. 2. Scheme of unit cell.

$-c \le x \le c$, and the zero temperature is assumed on the remaining part of the boundary plane as well as on the lower boundary of the body.

The heat conduction in the composite coating will be formulated by using the relations of the homogenized model with microlocal parameters given in [1]. The paper [1] considered the same periodic structure of composite as the coating assumed above. The homogenized model given by Woźniak [2] for an arbitrary cases of periodic structures of thermoelastic composites was first adopted for periodically layered bodies in [3] and has been applied to a series of problems of heat conduction and thermoelasticity reassumed partially in [4,5]. In the paper [1] the homogenized model of the heat conduction in periodically composites, in which every unit cell is composed of four beams with rectangular cross-sections is presented. Below, we give the final form of the governing equations of the homogenized model, which will be applied to the description problem in the coating. Namely, for the stationary plane case the equation of heat conduction in the coating takes the form:

$$k_x T_{1,xx} + k_y T_{1,yy} = 0, \tag{2.1}$$

where T_1 is the macrotemperature in the coating (the averaged temperature, which is the approximation of temperature), and

$$k_x = \frac{\eta_2 k_1 k_2}{(1-\eta_1)k_1 + \eta_1 k_2} + \frac{(1-\eta_2)k_3 k_4}{(1-\eta_1)k_3 + \eta_1 k_4} > 0,$$
$$k_y = \frac{\eta_1 k_1 k_4}{(1-\eta_2)k_1 + \eta_2 k_4} + \frac{(1-\eta_1)k_2 k_3}{(1-\eta_2)k_2 + \eta_2 k_3} > 0,$$
$$\eta_1 = a_1/a,\ \eta_2 = b_1/b. \tag{2.2}$$

The heat flux vector in the i-th component of the composite layer, $i = 1,2,3,4$, is determined by the relations

$$\mathbf{h}_1 = [B_1 T_{1,x}; B_2 T_{1,y}; 0],$$
$$\mathbf{h}_2 = [B_1 T_{1,x}; B_3 T_{1,y}; 0],$$
$$\mathbf{h}_3 = [B_4 T_{1,x}; B_3 T_{1,y}; 0],$$
$$\mathbf{h}_4 = [B_4 T_{1,x}; B_2 T_{1,y}; 0], \tag{2.3}$$

where

$$B_1 = \frac{k_1 k_2}{(1-\eta_1)k_1 + \eta_1 k_2},\quad B_2 = \frac{k_1 k_4}{(1-\eta_2)k_1 + \eta_2 k_4},$$
$$B_3 = \frac{k_2 k_3}{(1-\eta_2)k_2 + \eta_2 k_3},\quad B_4 = \frac{k_3 k_4}{(1-\eta_1)k_4 + \eta_1 k_3}. \tag{2.4}$$

It can be underlined that the homogenized model satisfies the continuity conditions on the interfaces between the components of the coating layer. Moreover, for the required continuity conditions the heat flux vector averaged within framework of the unit cell is introduced:

$$\mathbf{q} = [k_x T_{1,x}; k_y T_{1,y}; 0]. \tag{2.5}$$

The averaged heat flux normal to the axis Ox given by (2.5) will be used to approximately satisfy the continuity condition between the homogeneous layer and the coating composite. The considered problem is described by Eq. (2.1) in the region $0 < x < h$, $y \in R$, the equation of heat conduction in the homogenous base:

$$T_{0,xx} + T_{0,yy} = 0, \tag{2.6}$$

and the following boundary conditions:

a) on the boundary planes

$$T_0(x,-H) = 0,\ x \in R,$$
$$T_1(x,h) = \theta(x)H(1-|x|),\ x \in R. \tag{2.7}$$