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Conjugated heat transfer in circular microchannels with slip flow and axial diffusion effects

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ABSTRACT

An integral transforms analysis is undertaken for conjugated heat transfer in circular microchannels with laminar gaseous flow in the slip flow regime. The solution methodology is based on the Generalized Integral Transform Technique applied to a single domain formulation that models the coupled heat transfer phenomena at the fluid stream and at the channel wall. The single domain formulation results in just one partial differential equation for the energy balance, making use of spatially variable coefficients with abrupt transitions, and accounting for the temperature jump at the interface due to the Knudsen numbers within the slip flow regime. This work extends the single domain formulation strategy, not a priori applicable to problems with discontinuities, by considering a very thin fictitious layer at the fluid-wall interface region, so as to mathematically represent an equivalence to the temperature jump. An integral balance technique for enhancing the convergence of the eigenfunctions is employed, so as to achieve more accurate results and improve convergence for the so derived multiscale problem. The results obtained are critically compared against a dedicated finite difference numerical solution for the original multi-region problem. Results for the Nusselt number are presented in order to investigate its behavior with respect to different Péclet and Knudsen numbers, and different wall thicknesses values, confirming the importance of the combined effects of slip flow, axial conduction and heat transfer conjugation in the analysis.

1. Introduction

Several earlier works in the analysis of thermal microsystems led to the observation of discrepancies between the experimental results and classical correlations or simulations for the associated heat transfer coefficients, as reviewed in [1]. These discrepancies are mainly related to the adoption of classical hypothesis employed in modeling macro-scale problems, which may no longer be valid when dealing with heat and fluid flow in microsystems [2]. In such cases, due to the very small characteristic lengths involved, it is required to modify and extend the flow and convective heat transfer modeling, in comparison to the usual simplified macro-scale formulations [3].

A few analytically based solutions have been provided in the literature aiming at the analysis of micro-scale convective heat transfer, such as for instance for heat transfer with slip flow in circular microtubes [4,5], rectangular and parallel plates microchannels [6–8], and the investigation of viscous heating and fluid property variation [9]. Nevertheless, the effects of axial diffusion due to the low Péclet numbers involved and the conduction-convection conjugation effect due to the microsystem substrate, have been avoided in such analytical developments. In ref. [10] the conjugation effect was taken into consideration for heat transfer in a parallel plates microchannel, employing the Generalized Integral Transform Technique (GITT) [11,12] in combination with a single domain formulation. This reformulation strategy was proposed towards rewriting multi-region problems, such as those in conjugated conduction-convection heat transfer, into single region problems with space variable thermophysical properties and source terms, allowing for a single integral transformation operation over the

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Nomenclature

$c_{p,\,f}$	fluid specific heat at constant pressure
$c_{v,\,f}$	fluid specific heat at constant volume
h_z	local heat transfer coefficient
k	thermal conductivity
K	dimensionless thermal conductivity
Kn	Knudsen number
K_{fic}	dimensionless thermal conductivity of the fictitious layer
L_z	microchannel length
L_{fic}	dimensionless thickness of the fictitious layer
M	truncation order of the eigenfunction expansion (eigenvalue problem solution)
N	truncation order of the temperature eigenfunction expansion
Nu	Nusselt number
N_ψ	norm corresponding to temperature eigenfunction, ψ
N_Ω	norm corresponding to auxiliary eigenfunction, Ω
Pe	Péclet number
Pr	Prandtl number
q_w	wall heat flux
r	radial position
R	dimensionless radial position
Re	Reynolds number
T	temperature
T_w	prescribed temperature at the external wall
T_in	prescribed temperature at the inlet
u	velocity field
z	longitudinal position
Z	dimensionless longitudinal position

Greek letters

α_m	tangential momentum accommodation coefficient
α_t	thermal accommodation coefficient
β_t	wall temperature jump coefficient
β_v	wall velocity slip coefficient
γ	specific heat ratio
λ	molecular mean free path
η	eigenvalues corresponding to auxiliary eigenfunction, Ω
Ω	auxiliary eigenfunctions
μ	eigenvalues corresponding to temperature eigenfunction, ψ
ψ	temperature eigenfunctions
ρ	specific mass
θ	dimensionless temperature

Subscripts and superscripts

av	average
f	quantities related to the fluid region
fic	quantities related to the fictitious layer
o	quantities at the outer wall
s	quantities related to the solid region
i	quantities at the inner wall
j, l, m, n	order of eigenquantitites
*	domain including the fictitious layer
~	normalized eigenfunction
–	integral transform

whole physical domain. This hybrid approach has also been demonstrated quite successfully in dealing with axial diffusion effects [13], complex configurations and irregular regions [14–16], as reviewed in [3], automatically satisfying heat flux and temperature continuity conditions at the interfaces, and without the need for domain decomposition schemes. Nonetheless, rarefaction effects were not included in those conjugated heat transfer studies [10,13–16].

Hence, the present work addresses an analytically based solution extending the analysis performed in [10,13–16] in order to simultaneously handle all these typical micro-scale effects: (i) the slip boundary condition in opposition to the classical no-slip boundary condition, together with the interfacial temperature discontinuity (temperature jump condition); (ii) the inclusion of the axial conduction term in the fluid energy equation; and (iii) the consideration of the conduction-convection conjugation effect. Despite the importance of modeling and analytically handling these three extensions of the classical Graetz problem at the microscale, the analysis of their combined effects has been somehow overlooked in the earlier literature, with a few relevant exceptions that employed discrete numerical methods and included simultaneously developing flow conditions [17,18].

Actually, in ref. [19] the combination of the single domain formulation and the integral transforms approach was first extended to solve the conjugated heat transfer problem within the slip flow regime, when a fictitious very thin layer between the fluid region and the actual channel wall was introduced, in order to impose the desired thermal resistance between the fluid and the wall, and thus model the temperature jump at the interface. This fictitious solid layer needs to be kept much thinner than the characteristic length of the channel, so as to minimize the perturbation to the original problem geometry. However, such a solution based on the eigenfunction expansion of the temperature field, obtained through the integral transformation of the heat transfer problem, ends up by involving the solution of an eigenvalue problem with abrupt and multiscale variations on the governing space variable coefficients in the single domain formulation, resulting in undesirable slower convergence rates.

Therefore, based on a very recent development regarding the convergence acceleration of eigenfunction expansions for Sturm-Liouville problems [20,21], employing an integral balance analytical procedure [11,12], the present work proposes the combination of the single domain formulation and the integral transforms method to accurately and efficiently handle the conjugated heat transfer problem within circular microchannels in the slip flow regime, including axial diffusion effects. For verification purposes, a steady or quasi-steady state test problem with hydrodynamically fully developed and thermally developing flow is addressed. A dedicated finite difference simulation is also implemented for the original multi-region problem [22], allowing for critical comparisons. In order to investigate the influence of the microscale effects on convective heat transfer, the Nusselt number is computed for different combinations of wall thicknesses, Péclet and Knudsen numbers.

2. Problem formulation and solution methodology

Consider the incompressible gas flow within a circular microchannel with length L_z, cross-section with inner radius r_i and outer radius r_o, as illustrated in Fig. 1, undergoing convective heat transfer due to a prescribed temperature T_w at the external wall, different from the inlet temperature T_{in}. The channel wall is considered to participate