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On identifying optimal heat conduction topologies from heat transfer paths analysis

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ABSTRACT

Making clear how thermal loads are equilibrated and transferred through a structure is of importance for identifying optimal heat conduction topologies in problems of cooling scheme design. This paper proposes a new method for computing the orientations and locations of heat transfer paths from a standard finite element analysis. First, a non-dimensional parameter (C^*) is introduced, based on the variation in thermal compliance of the structure, to quantify the contribution of an arbitrary point in the structure to heat conduction performance. Then, a plotting algorithm is presented to search for the ridge lines of the calculated C^* curved surface, which provide the steepest ascent in C^* values from the points of heat sink to the points of heat source. Based on this, mathematical conditions (termed uniformity and continuity requirements) are discussed in detail and utilized as new criteria for tailoring the layouts of heat transfer paths in order for conductive cooling to be more effective. Finally, the derived criteria are incorporated into a commonly used topology optimization algorithm (SIMP algorithm), and validated though several benchmark cases. Unlike the conventional methods, the suggested approach optimizes only the orientations and locations of heat transfer paths and does not intentionally concern the overall temperature distribution, the application of which can potentially lead to worthwhile improvements at the conceptual stage of cooling design processes.

1. Introduction

Conductive heat transfer is widely used in the cooling of electronic equipment. The design of conductive cooling schemes has been approached by various methods, in which the optimization of heat transfer paths is of particularly interest, especially when the volume to be cooled has a low thermal conductivity [1–3]. However, to be able to accommodate higher and higher effective power densities within a given volume, how to realize the most efficient heat transfer paths is becoming a challenging problem for designers. In many instances, optimized cooling systems have incredible resemblances with respect to shapes and topologies found in both living and nonliving natural structures [4–6]. In the past, approaches such as constructal theory were used in an attempt to explain such resemblances and to obtain natural-appearing structures for different applications in heat conduction [7,8]. With the predefined simplified building blocks, a tree-like thermal network was configured from small to large through several levels of assembly. Although such method is very easy to be implemented, the heat transfer paths bifurcate regularly and rigidly, which may be geometrically far from the true optimum or close-to-optimum layouts. With the increasing variety of electronic chips and diversities of heat dissipating devices, flexible design of heat transfer paths is expected.

Another way to obtain cooling design solutions is to apply topology optimization which follows the simple idea of searching for the best material layout or distribution in order to improve certain performance of the structure [9,10]. In recent decades, topology optimization has been broadened to different branches of applications including heat conduction scenarios, offering the greatest potential for exploring ideal and optimum heat transfer paths. In general, there are two major categories of computational framework for topology optimization. One is the microstructure based approach, in which the conduction domain was discretized into finite elements and certain optimality criteria was applied to find elementwise size or density distributions, like the homogenization method (HDM) [11], the variable thickness method (VTM) [12], and the solid isotropic material with penalization (SIMP) method [13–15]. The other is the geometry based approach, in which the structural boundaries were represented by certain geometric functions, and the outlines of targeting structures were changed by updating the geometric functions during the optimization process, such as the

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level set (LST) method [16,17], the evolutionary structural optimization (ESO) method [18], the bidirectional ESO (BESO) method [19], the phase-field method [20], the topological derivative method [21], and so on. More details about the topology optimization for heat conduction design can be found in literature [22].

Although the aforementioned researches are growing fast and essentially mature, there is little available on how to characterize and quantify the trajectory and efficiency of heat conduction along the heat transfer paths. So far, in the heat transfer community, there is no generally accepted approach or method to visualize the heat transfer path [23,24]. However, designers need to clarify how heat is equilibrated and transferred through the structure, as it is important to make sure that the structure will perform its intended heat conduction function properly even under various unforeseen damage conditions. From this point of view, a successful identification of optimal heat conduction topologies would require deep insights into the interpretation of heat transfer paths, which includes a well-established theory to quantify the heat transfer paths and an efficient plotting algorithm to visualize and tailor the heat transfer paths. With all these taken into consideration, this paper gives a quantitative measure of the contribution of any point in the structure to heat conduction performance. Such quantitative information will facilitate designers to suppress the redundant features and finally realize the optimum material utilization. The proposed approach optimizes only the orientations and locations of heat transfer paths and does not intentionally concern the overall temperature distribution, which can be considered as a major difference compared with the conventional optimization methods.

This paper is organized as follows. We first introduce the fundamental theory of heat transfer paths in Section 2. Mathematical properties of the path plots are then discussed in detail. A new evolutionary algorithm is developed for the identification of heat conduction topologies. Key details about the algorithm are provided in Section 3. Numerical studies are presented in Section 4. The main findings of the research work are concluded in Section 5.

2. Fundamental theory

2.1. Definition of the non-dimensional parameter C^*

Assume that there is a heat conduction domain (Ω), as shown in Fig. 1. We focus on three kinds of points, Point A (Heat sink), Point B (Heat source) and Point C (arbitrary point). The Dirichlet boundary Γ_T (i.e., temperature boundary, T_0) is specified at the Point A and the Neumann boundary Γ_q (i.e., heat flux boundary, q'') is specified at the Point B. When the temperature degree of freedom at Point C is unconstrained or free, the steady-state temperature distribution over the domain Ω is solved and denoted as $\mathbf{T}$. The thermal compliance of the conduction domain is denoted as ψ,

$$\psi = \frac{1}{2}\mathbf{T}^{\mathrm{T}}\mathbf{K}\mathbf{T} \tag{1}$$

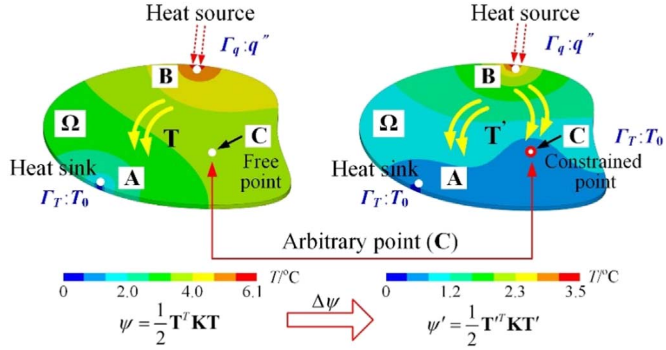

Fig. 1. Definition of the non-dimensional parameter C^*.

where, $\mathbf{K}$ is the global conductivity matrix of the conduction domain.

When the temperature degree of freedom at Point C is fixed (set to T_0), the steady-state temperature distribution over the domain Ω is solved again. As a result, the temperature distribution of the conduction domain changes from $\mathbf{T}$ to $\mathbf{T}'$, and the thermal compliance turns into ψ',

$$\psi' = \frac{1}{2}\mathbf{T}'^{\mathrm{T}}\mathbf{K}\mathbf{T}' \tag{2}$$

We introduce a non-dimensional parameter C^*, which is the scalar quantity of the variation in thermal compliance. The value of C^* at any point of the conduction domain can be obtained by Eq. (3) and the distribution of C^* is termed as C^* field.

$$C^* = 1 - \frac{\psi'}{\psi} \tag{3}$$

Since the thermal compliance (ψ') has a lower value when the connectivity between the heat source (Point B) and the constrained point (Point C) is stronger, the C^* value can be used to quantify the contribution of an arbitrary point in the structure to the overall heat conduction performance. That is, the greater the value of C^*, the stronger the heat conduction capacity between the heat source point and the arbitrary point. The computation of the C^* field requires a set of reanalysis, which means the nodes in the conduction domain will be fixed (i.e., the nodal temperature is set to T_0) one by one during the iterative computation. In this end, the number of reanalysis is equal to the number of nodes in the conduction domain.

2.2. Plotting algorithm of heat transfer paths

After characterizing the C^* field, the gradients of such scalar field are defined as the conductivity lines, which give an indication on how the heat is transferred through the structure. These conductivity lines can also be regarded as the orthogonal lines with respect to the potential lines of the C^* curved surface. The conductivity line that provides the steepest ascent in C^* values from the points of heat sink to the points of heat source is defined to be the heat transfer path in the structure. From another point of view, the heat transfer path is the successively traced line along the largest gradient $\mathbf{p}$ of the C^* curved surface, which is expressed as following.

$$\mathbf{p} = -\mathrm{grad}\, C^* \tag{4}$$

We make a simple analogy between the natural mountain and the calculated C^* curved surface, as shown in Fig. 2. The heat transfer path can be considered as the ridge line of the C^* curved surface, which is the most effective route for heat conduction in domain Ω.

From the above analysis, the heat transfer paths can be obtained through successive trace along the direction of the largest gradient vector. In this research, the commonly used section analysis method is applied to plot the heat transfer paths [25]. The point with the maximum C^* value on the curve of the section is defined as the ridge point. The ridge lines can be obtained by sequential connection of these ridge points. The plotting algorithm of heat transfer paths mainly includes three basic steps.

Step_1: Static calculations are performed to get the nodal temperatures of the FE model. The value of C^* at each node point is then calculated by substituting the results into Eq. (3).
Step_2: The potential lines of C^* are obtained mathematically by means of the spline function, and the orthogonal lines to these potential lines are defined as the conductivity lines.
Step_3: The section analysis is applied to get the ridge points of the C^* curved surface, which is started from the point of heat sink (point A) and implemented iteratively along the calculated potential lines. The heat transfer paths are then obtained through the sequential connection of these ridge points.