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Joule heating and viscous dissipation in flow of nanomaterial by a rotating disk

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ABSTRACT

This paper aims to analyze the flow of second grade nanoliquid by a rotating disk. Nanofluid under investigation strongly depends upon Brownian motion and thermophoresis. Heat transfer is studied subject to dissipation and Joule heating. Governing problems are made dimensionless. After this the out coming problems for momentum, temperature and concentration are solved. The convergence criteria related to solutions is spelled out. Convergence interval for solutions is analyzed. Impact of different variables on velocity, concentration and temperature is elucidated by plotting graphs. Velocity and temperature gradients are calculated and discussed. The obtained results demonstrate that velocity field enhances for larger estimation of viscoelastic variable. Further results also demonstrate that velocity gradient has opposite effects for Hartman number and viscoelastic parameter. Temperature gradient is more for higher estimation of Reynolds number.

1. Introduction

Flow due to rotating disk is significant in different fields relating to space study, fluid dynamics and geophysics. Rotating flow problem has great importance in various applications like food processing, rotor-stator system, chemical mixing chambers, geothermal extraction, oil recovery process, power generators *etc.* Mustafa [1] investigated MHD nanomaterial partial slip flow by a rotating disk. Axisymmetric flow by a rotating disk with uniform/oscillatory suction is studied by Chawla et al. [2] MHD force convective flow of non-Newtonian liquid due to rotating disk with thermophoretic particle deposition is explored by Doh and Muthtamilselvan [3]. Two-layer unsteady viscous liquid film flow due to rotating disk is analyzed by Dandapat and Singh [4]. They used asymptotic method for small estimation of Reynolds number. Consequences of homogeneous and heterogeneous reactions flow of nanomaterial by a rotating disk with variable thickness is presented by Hayat et al. [5]. Flow of Ostwald-de Waele liquid due to rotating disk of variable thickness is examined by Xun et al. [6]. Convective flow of viscoelastic nanomaterial by two rotating stretchable disks is investigated by Hayat et al. [7]. MHD stagnation point flow of magnetite (Fe_3O_4), Mn-Zn ferrite ($Mn - ZnFe_2O_4$) and cobalt ferrite ($CoFe_2O_4$) due to rotating disk is examined by Mustafa et al. [8]. Radiative flow of viscous liquid due to rotating disk with variable thickness is studied by Hayat et al. [9]. Non-Fourier heat flux in flow saturating porous space by rotating disks is explored by Hayat et al. [10].

The non-Newtonian materials in past few years have attracted the attention of researchers. It is because of their wide applications in different fields like crude oil extraction from petroleum product, processing of food stuff, fiber coating and paper production *etc.* The non-Newtonian materials cannot be investigated through the customary Navier-Stokes expressions. Besides this a constitutive equation for description of all non-Newtonian materials is inadequate. Therefore many non-Newtonian models for example Maxwell, Jeffrey, Cross, second grade *etc.* have been considered. The second grade model [11] is essential for portraying the effects of normal stress. However this model does not describe the shear thinning and shear thickening effects. Several scientists have worked for flows of second grade fluids [12–20].

This article presents the flow of second grade liquid due to rotating disk accounting dissipation and Joule heating. Uniform applied magnetic field is also taken into account. HAM [21–35] is adopted to get the convergence of series solutions. Impact of different variables on velocity, concentration and temperature is examined and discussed graphically. Velocity and temperature gradients are computationally analyzed in Tables 2 and 3.

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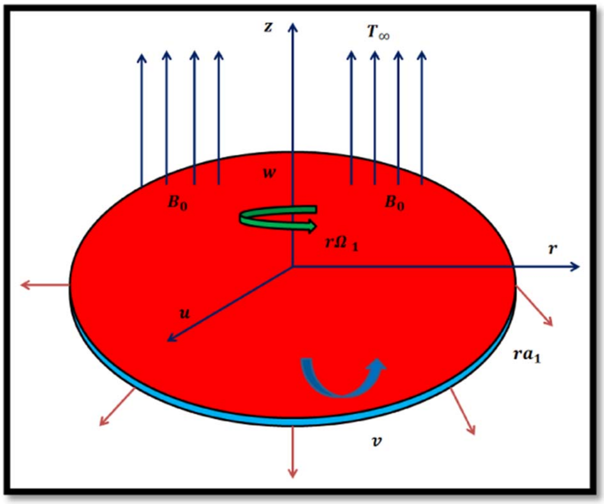

Fig. 1. Physical model.

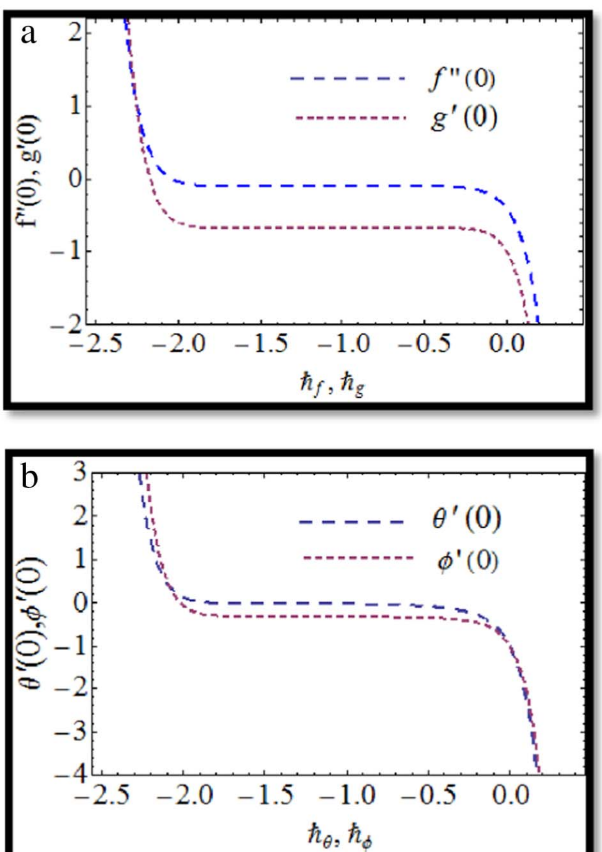

Fig. 2. (a): *h – curves for f″ (0) and g′ (0).*
(b): *h – curves for θ(0) and ϕ′ (0).*

2. Problem statements

Let us consider three-dimensional steady flow of second grade nanomaterial by a rotating disk. Interpretation of nanofluid under consideration strongly depends upon the Brownian movement and thermophoresis.

Heat transfer process is explored subject to dissipation, heat generation/absorption and Joule heating. Uniform magnetic field is applied in $z-$ direction with strength B_0. The disk rotates with constant velocity Ω_1 and a_1 stretching rate. Radial direction is along $r-$ axis and

Table 1
HAM solutions convergence when $\mathrm{Re} = 0.3$, $\mathrm{Pr} = 1.5$, $Sc = 1$, $Nt = 0.01$, $Nb = 0.3$, $Ec = 0.4$, $M = 0.4$, $\beta = 0.2$ and $A = 0.4$.

Order of approximations	$-f''(0)$	$-g(0)$	$-\theta(0)$	$-\phi(0)$
1	0.2365	0.7967	0.3427	0.4280
5	0.0968	0.6761	0.0799	0.3272
10	0.0850	0.6793	0.0223	0.3047
15	0.0853	0.6809	0.0078	0.2993
20	0.086324	0.99843	0.46443	0.41518
20	0.0855	0.6809	0.0045	0.2973
26	0.0855	0.6808	0.0038	0.2963
27	0.0855	0.6808	0.0038	0.2962
30	0.0855	0.6808	0.0038	0.2959
31	0.0855	0.6808	0.0038	0.2959

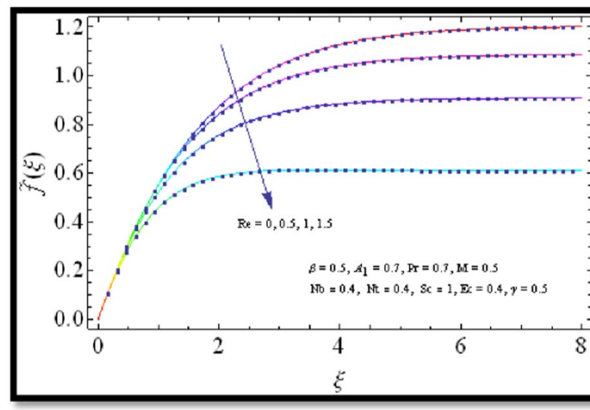

Fig. 3. *Re via f(ξ).*

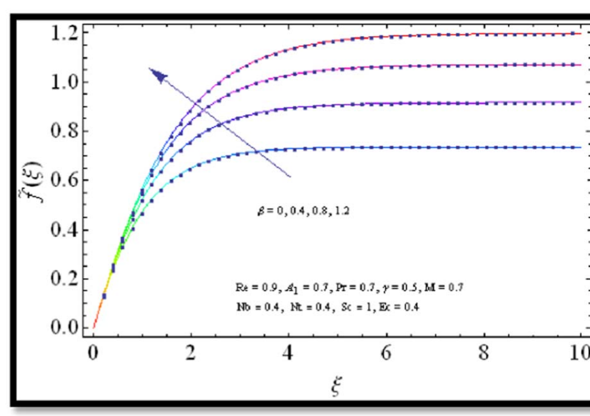

Fig. 4. *β via f(ξ).*

tangential direction along $z-$ direction (see Fig. 1). After applying the boundary layer approximation the law of conservation of mass, momentum, energy and concentration take the form:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = \upsilon\frac{\partial^2 u}{\partial z^2} + \alpha_1\left(\begin{array}{c} u\frac{\partial^3 u}{\partial r\partial z^2} + w\frac{\partial^3 u}{\partial z^3} + \frac{\partial^2 v}{\partial z^2}\frac{\partial v}{\partial r} - \frac{v}{r}\frac{\partial^2 v}{\partial z^2} \\ + \frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial r\partial z} + \frac{\partial^2 u}{\partial z^2}\frac{\partial w}{\partial z} + 3\frac{\partial^2 u}{\partial r\partial z}\frac{\partial u}{\partial z} + 2\frac{\partial u}{\partial r}\frac{\partial^2 u}{\partial z^2} - \frac{1}{r}\left(\frac{\partial u}{\partial z}\right)^2 \end{array}\right) - \frac{\sigma}{\rho}B_0^2 u, \tag{2}$$

$$u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = \upsilon\frac{\partial^2 v}{\partial z^2} + \alpha_1\left(\begin{array}{c} u\frac{\partial^3 v}{\partial r\partial z^2} + w\frac{\partial^3 v}{\partial z^3} - \frac{\partial v}{\partial z}\frac{\partial^2 u}{\partial r\partial z} \\ + \frac{u}{r}\frac{\partial^2 v}{\partial z^2} - \frac{\partial v}{\partial z}\frac{\partial^2 u}{\partial r\partial z} - \frac{1}{r}\frac{\partial u}{\partial z}\frac{\partial v}{\partial z} \end{array}\right) - \frac{\sigma}{\rho}B_0^2 v, \tag{3}$$