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Generating optimal topologies for heat conduction by heat flow paths identification☆

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ABSTRACT

Knowing how heat "flows" inside a structure is significant for designers to make sure that the structure will perform its intended heat conduction function properly. This paper describes the principle and the process of defining an optimal topology for heat conduction by interpreting the heat flow paths obtained from a standard finite element analysis. First, a simple equilibrium equation is introduced to characterize heat flow paths along which the heat transfer rate remains constant as it traverses the conduction domain. Then, mathematical properties of the path plots are discussed in detail, providing designers with an insight of what kind of path distribution is the best for heat conduction. A brand new optimizer is developed to sketch the most effective topology by homogenizing the spacing of heat flow paths across the conduction domain. Finally, the optimization procedure is illustrated in a directive and descriptive way, where the well-known dichotomic constructal tree is selected as the initial layout to be tested. Unlike the conventional methods, the new approach needs neither the sensitivity analysis nor the modification of the existing finite element programs, it is very straightforward to use in practice.

1. Introduction

In electronic engineering or micro-electronic instruments, the issue of how to effectively cool the heat generating volume or surface has attracted more and more attention. One common and cost-effective solution is based on inserting certain quantity of high conductivity material in the heating region to collect and discharge the generated heat to a heat sink by heat conduction. The cooling effectiveness is dependent not only on the quantity and quality of high conductivity material but also on the topological arrangement of high conductivity link inside the heat generating region. At this background, the planning of heat conduction paths is a fascinating problem for designers. However, it is also a difficult problem to fully harness the cooling potential of the added high conductivity material, especially when the thermal loading condition is complicated.

When pressed with engineering puzzles, human often draw guidance and inspiration from the natural world. A well-known case is the constructal theory proposed by Bejan [1–3], in which tree-like networks are assembled iteratively to form an optimal configuration so that the overall thermal resistance is minimized for heat conduction. Despite its great application potential, most of the cooling channels predicted from the constructal theory bifurcate regularly and rigidly, which may be geometrically far from the true optimal layout. With the increasing variety of electronic chips and diversities of cooling requirements, flexible design of heat conduction paths is needed. Since the idea of layout optimization has been presented as a problem of searching for the optimal material distribution [4,5], topology optimization has become the most powerful tool for structure design. In recent decades, topology optimization has been developed for more and more complex scenarios. In general, there are two broad categories of topology optimization used for heat conduction design. One is the microstructure based approach, like the homogenization design method (HDM) [6,7], the solid isotropic microstructures with penalization (SIMP) method [8–10], etc. The other is the geometry approach, like the evolutionary structural optimization (ESO) method [11,12], the bidirectional ESO (BESO) method [13], the level set (LST) method [14–16], the cellular automation (CA) method [17], and so on. More details about the topology optimization for heat conduction design can be found in [18].

Although the literature on heat conduction optimization and the technologies that underpin it are growing fast, there is little available on how a designer creates the conduction topology by optimizing the distribution pattern of heat flow paths. In fact, a well-designed heat flow paths can deliver better topologies for heat conduction. The successful analysis on heat flow paths in topology optimization would

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require a well-established supporting theory, an efficient path-plotting strategy and a proficient topology identification algorithm. With all these taken into consideration, this paper addresses the creation of heat conduction topology from a different perspective, the perspective of heat flow paths identification. The design idea takes inspiration from an analogy of streamlines in hydromechanics [19,20]. Such a simple analogy appears to provide insights on the way of how a structure conducts heat, and facilitate the suppression of redundant features in topology optimization, which can be considered as a great improvement compared with other optimization methods.

The layout of this paper is as follows. We first describe the fundamental theory of heat flow paths in Section 2. Mathematical properties of the path plots are then explained in detail. A new evolutionary algorithm is developed for heat conduction design. Key details about the algorithm are provided in Section 3. Numerical examples are presented in Section 4. The main findings of the research work are concluded in Section 5.

2. Fundamental theory

Suppose that an arbitrary linear thermal conductor occupies domain Ω, where Dirichlet boundary $\boldsymbol{\Gamma_T}$ (i.e., temperature boundary, T_0) and Neumann boundary $\boldsymbol{\Gamma_q}$ (i.e., heat flux boundary, q) are pre-specified. A heat 'stream tube' is introduced, which inherits properties similar to that of a fluid stream tube, for instance, no heat (fluid) should cross the tube boundaries such that the equilibrium is satisfied over the tube.

$$\Phi_a = \Phi_b \tag{1}$$

where Φ_a and Φ_b are the heat transfer rate at the section AC and BD, respectively.

An arbitrary segment of unit area taken from the tube boundary is shown in Fig. 1, in which the vector $\boldsymbol{n}$ is the normal vector of the boundary and is given by

$$\boldsymbol{n} = n_x \cdot \boldsymbol{i} + n_y \cdot \boldsymbol{j} + n_z \cdot \boldsymbol{k} \text{ and } n_x^2 + n_y^2 + n_z^2 = 1 \tag{2}$$

where $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ are the unit vector in the X, Y, and Z directions, respectively.

The heat flux $\boldsymbol{Q}$ at every point in the domain Ω is expressed as

$$\boldsymbol{Q} = q_x \cdot \boldsymbol{i} + q_y \cdot \boldsymbol{j} + q_z \cdot \boldsymbol{k} \tag{3}$$

where q_x, q_y, and q_z are the value of heat flux in the X, Y, and Z directions, respectively.

The heat transfer rate at the arbitrary plane can be obtained by integrating the heat flux:

$$\Phi = \int \boldsymbol{Q} \cdot \boldsymbol{n}\, dA \tag{4}$$

where the dot indicates the vector dot product.

The requirement for defining the heat flow paths is actually to create a heat tube in which there is no contribution to the heat transfer rate on its boundaries AB and CD. For boundary AB, this requires

$$\int_A^B \boldsymbol{Q} \cdot \boldsymbol{n}\, dA = 0 \tag{5}$$

That is,

$$\int_A^B \left(\begin{bmatrix} q_x & q_y & q_z \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{i} \\ \boldsymbol{j} \\ \boldsymbol{k} \end{bmatrix} \right) \cdot \left(\begin{bmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \end{bmatrix} \cdot \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \right) dA = 0 \tag{6}$$

Eq. (6) is achieved if the tangent lines of every points on boundaries AB and CD are parallel to the corresponding heat flux vector $\boldsymbol{Q}$. This means the heat flow paths can be obtained by tailoring the local angular orientation of the heat tube boundaries. A numerical integration scheme is employed to plot the contours through the pointing vector field. Unlike the commonly used fourth-order Runge–Kutta algorithm, the approach used in this research is not time independent but space independent, which means the time discretization, Δt, is replaced by the scalar spatial discretization, Δl, in order to represent a small increment along the heat flow path, as shown in Eq. (7):

$$\boldsymbol{V} = \frac{\partial x}{\partial l} \cdot \boldsymbol{i} + \frac{\partial y}{\partial l} \cdot \boldsymbol{j} + \frac{\partial z}{\partial l} \cdot \boldsymbol{k} \tag{7}$$

where $\boldsymbol{V}$ is the pointing vector field defined over the conduction domain Ω.

The field of pointing vectors can be first defined by averaging the elemental values of heat flux onto the nodes embedded in the finite element mesh. Then the vector at any arbitrary point in the domain Ω can be obtained by relating the point with an element and then interpolating from the corresponding nodal values. Suppose that $P_i(x_i, y_i, z_i)$ is the initial point, a new point $P_{i+1}(x_{i+1}, y_{i+1}, z_{i+1})$ can be positioned by the modified Runge–Kutta scheme. The coordinates of the new point $\zeta_{i+1}(\zeta = x, y, z)$ is a weighted sum of ζ_i and $d\zeta_j$ $(j = 1, 2, 3, 4)$. There are four values of $d\zeta_j$; each new value is calculated at a location that depends on the previous one, as shown in Eqs. (8) and (9):

$$\begin{aligned} d\zeta_1 &= \left.\frac{\partial \zeta}{\partial l}\right|_{\zeta_n} \Delta l \quad & d\zeta_2 &= \left.\frac{\partial \zeta}{\partial l}\right|_{\zeta_n + \frac{d\zeta_1}{2}} \Delta l \\ d\zeta_3 &= \left.\frac{\partial \zeta}{\partial l}\right|_{\zeta_n + \frac{d\zeta_2}{2}} \Delta l \quad & d\zeta_4 &= \left.\frac{\partial \zeta}{\partial l}\right|_{\zeta_n + d\zeta_3} \Delta l \end{aligned} \quad \zeta = (x, y, z) \tag{8}$$

$$\zeta_{i+1} = \zeta_i + \frac{1}{6}(d\zeta_1 + 2d\zeta_2 + 2d\zeta_3 + d\zeta_4) \quad \zeta = (x, y, z) \tag{9}$$

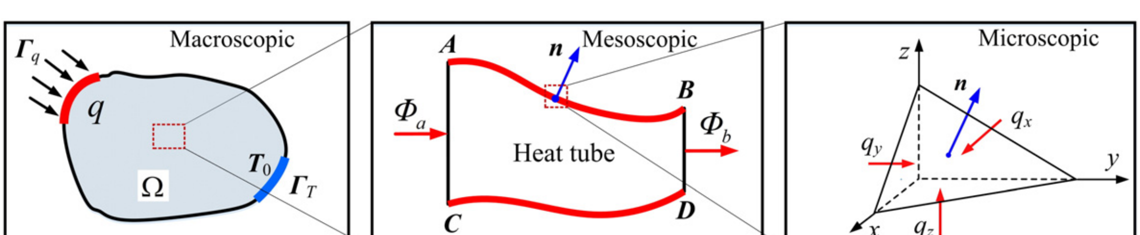

Fig. 1. Construction of the heat flow paths.