International Communications in Heat and Mass Transfer xxx (2015) xxx–xxx

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

# Transient stage comparison of Couette flow under step shear stress and step velocity boundary conditions☆

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ARTICLE INFO

Available online xxxx

*Keywords:*
Couette flow
Boundary conditions
Step wall shear stress
Step wall velocity

ABSTRACT

Couette flow has been widely used in many industrial and research processes, such as viscosity measurement. For the study on thixotropic viscosity, step-loading, which includes (1) step shear stress and (2) step velocity conditions, is widely used. Transient stages of Couette flow under both step wall shear stress and step wall velocity conditions were investigated. The relative coefficient of viscosity was proposed to reflect the transient process. Relative coefficients of viscosity, dimensionless velocities and dimensionless development times were derived and calculated numerically. This article quantifies the relative coefficients of viscosity as functions of dimensionless time and step ratios when the boundary is subjected to step changes. As expected, in the absence of step changes, the expressions reduce to being functions of dimensionless time. When step wall shear stresses are imposed, the relative coefficients of viscosity change from the values of the step ratios to their steady-state value of 1. But with step-increasing wall velocities, the relative coefficients of viscosity decrease from positive infinity to 1. The relative coefficients of viscosity increase from negative infinity to 1 under the step-decreasing wall velocity condition. During the very initial stage, the relative coefficients of viscosity under step wall velocity conditions are further from 1 than the one under step wall shear stress conditions but the former reaches 1 faster. Dimensionless development times grow with the step ratio under the step-rising conditions and approach the constant value of 1.785 under the step wall shear stress condition, and 0.537 under the step wall velocity condition respectively. The development times under the imposed step wall shear stress conditions are always larger than the same under the imposed step wall velocity conditions.





## 1. Introduction

In fluid dynamics, Couette flow is the laminar flow of a viscous fluid in the space between two parallel plates, one of which is moving while the other one keeps static. The transient start-up of Couette flow between two parallel plates one of which moves suddenly under imposed constant velocity has been investigated by many researchers [1,2]. Bandyopadhyay and Mazumder [3] studied the longitudinal dispersion of passive contaminant in an incompressible fluid between a parallel plate channel oscillating wall velocities. Analytical solutions of transient Couette flows of Newtonian fluid with constant and time-dependent pressure gradients have been proposed by Mendiburu et al. [4]. But the boundary condition was always a constant velocity.

There have been other researches on unsteady Couette flow of non-Newtonian fluid or under special situations. Exact solutions of unsteady Couette flows of generalized Maxwell fluid with fractional derivative were obtained by Wang and Xu [5] and Tan et al. [6]. Theoretical analysis of the velocity field and stress field of generalized second order fluid was presented by Xu and Tan [7]. Analytic solutions for unsteady unidirectional flows of an incompressible second grade fluid with a free surface or a moving plate and the associated frictional forces were reported by Hayat et al. [8]. Unsteady Couette flows of a second grade fluid with space dependent viscosity, elasticity and density were investigated by Asghar et al. [9]. Velocity distributions for transient Couette flows of Oldroyed-B fluid between two infinite parallel plates have also been reported [10,11]. Lacaze et al. [12] presented solutions to describe the flows of a viscoplastic fluid subjected to constant and sinusoidal moving plate velocities in a relative wide cylindrical Couette device. Solutions for unsteady Couette flow through a porous medium or in a magnetic or electric field have also been reported [13–15]. Unsteady magnetohydrodynamics (MHD) Couette flow of a third-grade fluid in the presence of pressure gradient and Hall currents was discussed by Azram and Zaman [16]. Unsteady Couette flow through of a viscoelastic fluid through a parallel plate channel filled with a porous medium was studied by Attia et al. [17]. Makinde and Franks [18] studied transient

☆ Communicated by P. Cheng and W.Q. Tao.
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http://dx.doi.org/10.1016/j.icheatmasstransfer.2015.10.011
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**Nomenclature**

| | |
|---|---|
| $d$ | distance between the two plates (m) |
| $t$ | time (s) |
| $t^*$ | characteristic time, $t^* = d^2/\nu$ (s) |
| $\tilde{t}$ | dimensionless time, $\tilde{t} = t/t^*$ |
| $t_d$ | development time (s) |
| $\tilde{t}_d$ | dimensionless development time, $\tilde{t}_d = t_d/t^*$ |
| $u$ | fluid velocity (m/s) |
| $u_w$ | bottom wall velocity (m/s) |
| $\tilde{u}$ | dimensionless velocity distribution |
| $u_s$ | steady velocity distribution of stage (1) (m/s) |
| $u_d$ | velocity after the flow field has developed (m/s) |
| $u_0$ | bottom wall velocity at stage (1) (m/s) |
| $u_1$ | bottom wall velocity at stage (2) (m/s) |
| $U$ | velocity during the developing period (m/s) |
| $x$ | vertical spatial coordinate (m) |
| $\tilde{x}$ | dimensionless vertical spatial coordinate |
| *Greek symbols* | |
| $\beta_n$ | eigen values |
| $\varepsilon$ | step ratio |
| $\dot{\gamma}$ | shear rate ($s^{-1}$) |
| $\dot{\gamma}_d$ | developed shear rate ($s^{-1}$) |
| $\mu$ | calculated dynamic viscosity (Pa s) |
| $\mu_a$ | actual dynamic viscosity of fluid (Pa s) |
| $\tau$ | shear stress (Pa) |
| $\tau_w$ | bottom wall shear stress (Pa) |
| $\tau_0$ | bottom wall shear stress at stage(1) (Pa) |
| $\tau_1$ | bottom wall shear stress at stage(2) (Pa) |
| $\tau_d$ | developed shear stress (Pa) |
| $\xi_n$ | eigen values |
| $\nu$ | kinematic viscosity of fluid ($m^2/s$) |
| $\varphi$ | relative coefficient of viscosity (dimensionless) |
| $\varphi_\tau$ | $\varphi$ under imposed wall shear stress condition (dimensionless) |
| $\varphi_u$ | $\varphi$ under imposed wall velocity condition (dimensionless) |

reactive MHD Couette flow with temperature dependent viscosity and thermal conductivity. Boundary conditions of all above researches were either constant or time-dependent velocity at the moving wall. Imposed shear stress boundary conditions have never been considered.

There have been some researchers paying their attention on boundary conditions of controlled tangential surface force or torque. Ting [19] discussed unsteady Couette flows of second-order fluids with constant tangential surface force. Considering inertia of rotation, Ravey et al. [20] studied the transient motions of rotor based on an air-bearing viscometer filled with Newtonian fluids. Transient Couette flow of an Oldroyd fluid between two circular cylinders was investigated by Bernardin and Nouar [21]. The flow was induced by applying a constant torque to the inner cylinder with the outer cylinder kept stationary. Asymptotic analysis on acceleration and relaxation process near the inner cylinder was carried out to determine the time for inner cylinder and fluid to reach steady state.

Muzychka and Yovanovich [22] investigated the transient stage of Couette flow between two parallel plates and proposed a compact expression to describe the time-dependent shear stress on the internal surface of moving plate. Emin Erdoğan [2] concluded that the time required to attain the asymptotic values of the skin friction or volume flux of a Couette flow is $d^2/\nu$, where $d$ indicates the gap between the two parallel plates and $\nu$ represents the kinematic viscosity of the fluid. There have been several reasonable time scales for Taylor vortex flow to reach fully developed in cylindrical Couette flow, such as $t^* = d^2/\nu$, where $d$ indicates radial gap [23,24], $t^* = H^2/\nu$, where $H$ indicates gap length [25,26], and a mixed time scale based on both $H$ and $d$ [27,28], $t^* = Hd/\nu$. Another description of viscous time scale incorporating both the gap width and the distance between the endwalls of the system was recommended by Czarny and Lueptow [29].

Couette flow has been used as the fundamental method for the measurement of viscosity [22]. Viscosity of time-dependent viscous fluid, such as thixotropic fluid, needs to be measured and recorded continuously. Step-loadings, either step shear stress or step shear rate are widely used to study the viscosity of thixotropic fluid [30–33]. The velocity distribution is not steady immediately after step-changing of boundary conditions. Only when the velocity gradient of fluid becomes time-invariant, will the measured viscosity represent the actual one.

In this article, transient Couette flow of an incompressible Newtonian fluid between two infinite parallel plates subjected to both step wall shear stress and step wall velocity boundary conditions are analyzed. The times needed for the velocity fields to allow for proper viscosity measurements called the "development times" are presented. For quantitative analysis on transient stage, we define the relative coefficient of viscosity $\varphi(t)$ and dimensionless development time $\tilde{t}_d$. The potential errors on viscosity measurements under different loading conditions are quantified and compared by examining these two parameters.

The remainder of this article is divided into seven sections. The problem analyzed in this article is outlined in the next section. This is followed by the definitions of the dimensionless parameters used to identify the development times. Section 4 presents the governing equations, the initial condition and the boundary conditions for the problem described in Section 2. The velocity distributions are then listed in Section 5. The relative coefficient of viscosity and the dimensionless velocity are then obtained in Section 6. This is followed by discussion of the results. Some concluding remarks are given to conclude the article.

## 2. Problem description

When viscosity is measured in coaxial cylinder viscometer, fluid is kept in concentric cylindrical cylinders. One cylinder is kept stationary while the other rotates. This leads to an axisymmetric velocity distribution. Neglecting end effects, the velocity is a function of the radial coordinate. For the study on thixotropic viscosity, step-loading is widely used which includes (1) step shear stress and (2) step shear rate conditions. In a rheometer, shear rate is imposed by setting the rotational speeds of the measuring system [34]. So imposed shear rate condition can be analyzed as the imposed velocity condition. The flow in the annulus can be modeled using flow between two infinite parallel plates as the ratio of the annulus gap between inner and outer cylinders to the radius of the inner cylinder is small. The remainder of this article presents analyses of flow between two flat plates used to calculate viscosity.

Fig. 1 shows the schematic of the problems considered in this article. Couette flows between two parallel plates separated by a distance $d$ are analyzed. Transient laminar flows of an incompressible Newtonian fluid are induced by imposing different conditions on the bottom wall while the top wall remains stationary. The fluid is restricted to flow parallel to the two plates leading to a velocity field which is a function of vertical spatial coordinate $x$ and time $t$. The flow is steady under a specific wall shear stress $\tau_0$ ($\tau_0 \neq 0$) or velocity $u_0$ ($u_0 \neq 0$) in stage (1) and the velocity distribution could be described by

$$u_s(x) = \frac{\tau_0}{\mu}(d - x) \tag{1}$$

under the imposed wall shear stress condition and

$$u_s(x) = \frac{u_0}{d}(d - x) \tag{2}$$

under the imposed wall velocity condition. Here $u_s$ is the steady velocity of stage (1) and also the initial velocity of stage (2). At the beginning of



Please cite this article as: C. Lei, et al., Transient stage comparison of Couette flow under step shear stress and step velocity boundary conditions, Int. Commun. Heat Mass Transf. (2015), http://dx.doi.org/10.1016/j.icheatmasstransfer.2015.10.011