International Journal of Heat and Fluid Flow 73 (2018) 114–123

Contents lists available at ScienceDirect

International Journal of Heat and Fluid Flow

journal homepage: www.elsevier.com/locate/ijhff

Turbulence modelling for flows with strong variations in thermo-physical properties

Gustavo J. Otero R., Ashish Patel, Rafael Diez S., Rene Pecnik*

Process and Energy Department, Delft University of Technology, Leeghwaterstraat 39, 2628 CB Delft, Netherlands

ARTICLE INFO

Keywords:
RANS Turbulence modelling
Compressible flow
Varying properties
Semi-local scaling

ABSTRACT

This paper presents a novel methodology for improving eddy viscosity models in predicting wall-bounded turbulent flows with strong gradients in the thermo-physical properties. Common turbulence models for solving the Reynolds-averaged Navier–Stokes equations do not correctly account for variations in transport properties, such as density and viscosity, which can cause substantial inaccuracies in predicting important quantities of interest, for example, heat transfer and drag. Based on the semi-locally scaled turbulent kinetic energy equation, introduced in [Pecnik and Patel, J. Fluid Mech. (2017), vol. 823, R1], we analytically derive a modification of the diffusion term of turbulent scalar equations. The modification has been applied to five common eddy viscosity turbulence models and tested for fully developed turbulent channels with isothermal walls that are volumetrically heated, either by a uniform heat source or viscous heating in supersonic flow conditions. The agreement with results obtained by direct numerical simulation shows that the modification significantly improves results of eddy viscosity models for fluids with variable transport properties.

1. Introduction

Turbulence plays a vital role in heat transfer and skin friction across the boundary layer in wall bounded flows. For engineers, it is therefore of paramount importance to accurately model turbulence during the design process of any flow guiding devices, such as heat exchangers with strongly cooled or heated flows, rocket propulsion systems, combustion chambers with chemically reacting flows, or turbomachinery flows with unconventional working fluids. In all these applications, strong heat transfer causes large temperature gradients and consequently large variations in density, viscosity, thermal conductivity, heat capacity, etc., which alter the conventional behavior of turbulence. Despite decades of research, turbulent flows with variable thermophysical properties are still far from being understood. Accordingly, turbulence models for engineering applications with large heat transfer rates are not able to provide accurate results for Nusselt numbers, pressure losses, or any other quantities of interest.

In the past, experiments and direct numerical simulations (DNS) have been performed to study turbulent flows over a wide range of Reynolds numbers for boundary layers, channel, pipes, among others (Bradshaw, 1977; Duan et al., 2010; Lee et al., 2013; Modesti and Pirozzoli, 2016). However, these detailed measurements and simulations are limited to simple geometries, and as the Reynolds number increases, DNS become computationally more expensive. Because of this fact, turbulence models for simulations of the Reynolds-averaged Navier–Stokes (RANS) equations rely on a limited number of accurate data, and their development is additionally hampered by the lack of knowledge on how turbulence is affected by strong variations of thermophysical properties. Since almost all turbulence models have been developed for incompressible flows, several extensions to include compressible effects have been proposed in the past by (Huang et al., 1994; Sarkar et al., 1991; Zeman, 1993). For example, if the turbulent kinetic energy (TKE) equation is derived on the basis of the compressible Navier–Stokes equations, additional terms appear, i.e. pressure -work and -dilatation, dilatational dissipation, and additional terms related to fluctuations of density, velocity, pressure, etc. The modification of the TKE in flows with strong heat transfer has been attributed to these terms and according models have been proposed in the past Sarkar et al. (1991); Zeman (1993); Huang et al., (1995). Huang, Bradshaw, and Coakley (Huang et al., 1994), analyzed the log-layer behaviour of a compressible boundary layer using turbulence models and claimed that the model closure coefficients must be a function of mean density gradients to satisfy the law-of-the-wall obtained with the van Driest velocity transformation (Van Driest, 1951).

A different approach to sensitize turbulence models for compressible flows with large density variations, was proposed by Catris and

* Corresponding author.
E-mail address: r.pecnik@tudelft.nl (R. Pecnik).
URL: http://dutw1479.wbmt.tudelft.nl/~renep/ (R. Diez S.).

https://doi.org/10.1016/j.ijheatfluidflow.2018.07.005
Received 7 March 2018; Received in revised form 15 June 2018; Accepted 17 July 2018

Nomenclature

$\check{\nu}$	Spalart–Allmaras eddy viscosity
δ_v	Viscous length scale
δ_{ij}	Kronecker delta
γ	Heat capacity ratio
κ	Von Karman constant (= 0.41)
λ	Thermal conductivity
μ	Dynamic viscosity
μ_t	Eddy viscosity
ω	Specific turbulent dissipation
Φ	Volumetric source term
ρ	Density
σ	Model constant
τ	Shear stress
ε	Turbulent dissipation
c_p	Isobaric heat capacity
Ec_τ	Friction based Eckert number ($= u_\tau^2/(\widetilde{T}_w\widetilde{c}_{p,w})$)
f_x	External body force
H	Enthalpy
h	Characteristic length, half channel height
k	Turbulent kinetic energy ($= u_i''u_i''/2$)
M_τ	Friction based Mach number
p	Pressure
P_k	Production of turbulent kinetic energy
Pr	Prandlt number ($= c_p\mu/\lambda$)
Pr_t	Turbulent Prandlt number
R	Specific gas constant
Re_τ	Friction Reynolds number ($= u_\tau\rho_w h/\mu_w$)
$Re_\tau^\star$	Semi-local Reynolds number
Re_b	Bulk Reynolds number ($= u_b h\rho_b/\mu_w$)
T	Temperature
t	Time
u	Velocity
$u^\star$	Universal velocity transformation ($= \int_0^{u^{vD}}[1+(y/Re_\tau^\star)\partial Re_\tau^\star/\partial y]\partial u^{vD}$)
u^{vD}	Van Driest velocity transformation ($= \int_0^{(u/u_\tau)}\sqrt{\rho/\rho_w}\,\partial(u/u_\tau)$)
u_τ	Friction velocity ($= \sqrt{\tau_w/\rho_w}$)
$u_\tau^\star$	Semi-local friction velocity ($= \sqrt{\tau_w/\langle\rho\rangle}$)
x	Length
y^+	Locally scaled wall distance ($= y\ Re_\tau/h$)
$y^\star$	Semi-locally scaled wall distance ($= y\ Re_\tau^\star/h$)

Accents and subscripts

$\widetilde{\phi}$	Dimensional quantity
ϕ	Locally scaled quantity
$\widehat{\phi}$	Semi-locally scaled quantity
ϕ_b	Bulk quantity
ϕ_c	Quantity at the channel center
ϕ_w	Quantity at the wall

Averaging operators

$< \cdot >$	Reynolds averaged $\phi = \langle\phi\rangle + \phi'$ with $\langle\phi'\rangle = 0$
$\{\cdot\}$	Favre averaged $\phi = \{\phi\} + \phi''$ with $\langle\rho\rangle\{\phi\} = \langle\rho\phi\rangle$, $\{\phi''\} = 0$ and $<\phi''> \neq 0$

Aupoix, 2000. They used the formulation developed by Huang et al., 1994 for the closure coefficients, to modify the diffusion term of the turbulent dissipation transport equation. Additionally, they argued that the diffusion of TKE acts upon the energy per unit volume [$(kg\,m^2/s^2)/m^3$] of turbulent fluctuations, which can be expressed as ρk. The diffusion of TKE is therefore based on ρk, while the diffusion coefficient is divided by the density on the basis of dimensional consistency. Their approach improved eddy viscosity models for supersonic adiabatic boundary layer flows, without including the additional compressibility terms. However, these ad-hoc corrections to the TKE equations need to be assessed for a wide range of flows, including standard low-speed flows (Roy, C., Blottner, 2007) and free shear flows (Smits and Dussauge, 2006).

In this study, we analytically derive modifications of eddy viscosity models for flows with strong property variations, which are based on the fact that the “leading-order effect” of variable properties on wall bounded turbulence can be characterized by the semi-local Reynolds number only (Pecnik and Patel, 2017; Patel et al., 2016a). The developed methodology is generic and applicable to a wide range of eddy viscosity models. To demonstrate the improvement, we have applied the modifications to five different EVM from literature (Cess, 1958; Spalart et al., 1994; Myong and Kasagi, 1990; Menter, 1993; Durbin, 1995) and compared the results to direct numerical simulations of heated fully developed turbulent channel flows with varying thermo-physical properties (Patel et al., 2016a; Trettel and Larsson, 2016). Furthermore, the density corrections proposed by Catris and Aupoix (2000) has been considered as well. The matlab source code used in this paper and the DNS data from (Patel et al., 2016a) are available on GitHub (Pecnik et al., 2018).

2. SLS Turbulence modelling

The semi-local scaling (SLS) as proposed by Huang et al. in 1995 (Huang et al., 1995), is based on the wall shear stress $\widetilde{\tau}_w$ and on local mean (instead of wall) quantities of density and viscosity to account for changes in viscous scales due to mean variations in the thermo-physical properties. The aim of the SLS was to collapse turbulence statistics for compressible flows at high Mach numbers with those of incompressible flows. In the SLS framework, the friction velocity and viscous length scale are defined as $u_\tau^\star = \sqrt{\widetilde{\tau}_w/\langle\widetilde{\rho}\rangle}$ and $\delta_v^\star = \langle\widetilde{\mu}\rangle/\langle\widetilde{\rho}\rangle u_\tau^\star$, respectively, where $< \cdot >$ indicates Reynolds averaging. Accordingly, the semi-local wall distance can be defined as $y^* = \widetilde{y}/\delta_v^\star$ and the semi-local Reynolds number as,

$$Re_\tau^\star = \frac{u_\tau^\star\langle\widetilde{\rho}\rangle\widetilde{h}}{\langle\widetilde{\mu}\rangle} = \sqrt{\frac{\langle\widetilde{\rho}\rangle}{\widetilde{\rho}_w}}\frac{\widetilde{\mu}_w}{\langle\widetilde{\mu}\rangle}Re_\tau, \tag{1}$$

where $Re_\tau = u_\tau\widetilde{\rho}_w\widetilde{h}/\widetilde{\mu}_w$ and $u_\tau = \sqrt{\widetilde{\tau}_w/\widetilde{\rho}_w}$, are the conventional friction Reynolds number and friction velocity based on viscous wall units. In general, any flow variable can be non-dimensionalized using wall based units and semi-local units. This is outlined in more detail in Table 1. It is important to note, that the friction velocities of both scaling are related through the wall shear stress by $\widetilde{\tau}_w = \widetilde{\rho}_w u_\tau^2 = \langle\widetilde{\rho}\rangle u_\tau^{\star 2}$. This relation will be used frequently throughout the paper.

Instead of exclusively using the semi-local scaling to collapse turbulence statistics for compressible flows with different Mach numbers, Pecnik and Patel (2017) extended the use of the scaling to derive an alternative form of the TKE equation for wall-bounded flows with a strong wall-normal variations of density and viscosity. Starting from the semi-locally scaled non-conservative form of the momentum equations, and with the assumption that the wall shear stress $\widetilde{\tau}_w$ changes slowly in the streamwise direction, the SLS TKE equation reads,

$$t_\tau^\star\frac{\partial\{\hat{k}\}}{\partial\hat{t}} + \frac{\partial\{\hat{k}\}\{\hat{u}_j\}}{\partial\hat{x}_j} = \widehat{P}_k - \hat{\varepsilon}_k + \widehat{T}_k + \widehat{C}_k + \widehat{\mathcal{D}}_k, \tag{2}$$

with production $\widehat{P}_k = -\{\hat{u}_i''\hat{u}_j''\}\partial\{u_i^{vD}\}/\partial\hat{x}_j$, dissipation per unit volume $\hat{\varepsilon}_k = \langle\hat{\tau}_{ij}'\partial\hat{u}_i'/\partial\hat{x}_j\rangle$, diffusion (containing viscous diffusion, turbulent