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One-point and two-point statistics of homogeneous isotropic decaying turbulence with variable viscosity

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A B S T R A C T

The decay of homogeneous isotropic turbulence in a variable viscosity fluid with a viscosity ratio up to 15 is analyzed by means of highly resolved direct numerical simulations (DNS) at low Reynolds numbers. The question addressed by the present work is how quantities such as the kinetic energy and the associated dissipation rate, as well as the inter-scale transport mechanism of turbulence are changed by local fluctuations of the viscosity. The comparison is performed with respect to the decaying homogeneous isotropic turbulence with constant viscosity (CV), equal to the mean value of the variable viscosity (VV). From the one-point budget equation of the turbulent kinetic energy, it is shown that the mean dissipation rate is nearly unchanged by variable viscosity effects. This result is explained by a negative correlation between the local viscosity and the local velocity gradients. However, the dissipation is a highly fluctuating quantity with a strong level of intermittency. From a statistical analysis it is shown that turbulent flows with variable viscosity are characterized by an enhanced level of small-scale intermittency with respect to CV flow, which results in the presence of smaller length scales. The effect of variable viscosity on the turbulent cascade is analyzed by a budget equation for the velocity structure function. From DNS it is shown that viscosity gradients contribute to the inter-scale transport mechanism in the form of an inverse transport, where information propagates from the small scales to the large scales.

1. Introduction

Turbulent flows encountered in engineering and environmental applications are very often characterized by spatio-temporal fluctuations of viscosity, which results from variations of temperature or species composition. A prominent example from geophysical flows is the convection in the earth's mantle, where the viscosity decreases with temperature. An other important case is the turbulent mixing in combustion systems, where a concentration dependent viscosity may affect the efficiency of turbulent mixing.

Fully developed turbulence reveals a large range of length scales, varying from the so-called integral length scale l_t, at which large velocity fluctuations occur, down to the smallest scale, the so-called Kolmogorov or dissipation scale η, at which turbulent fluctuations are dissipated due to viscosity. According to Kolmogorov's first hypothesis (Kolmogorov, 1941a; Kolmogorov, 1941b), enunciated for fluids with homogeneous physical properties, the statistics of the smallest scales should be universal and depend only on two parameters, namely the viscosity ν and the mean energy dissipation rate $\langle\varepsilon\rangle$. Kolmogorov's second hypothesis postulates that large scales of the flow decouple from the smallest scales and should become independent of viscosity, provided that the Reynolds number is sufficiently high. However, numerous experimental and numerical studies have indicated that Kolmogorov's traditional view is a crude assumption and that large and small scale quantities are strongly coupled, cf. Sreenivasan and Antonia (1997), Warhaft (2000). This coupling is referred to as Finite Reynolds Number effect (FRN) and it has been widely shown that is flow specific Antonia et al. (2015). The situation is even more complex for turbulent mixing with local viscosity variations. One has to cope with a turbulence-scalar interaction which is two-fold: the fluid motions affect the scalar mixing, while mixing induced viscosity changes affect the dynamics of the velocity field.

Viscosity represents the most important property of turbulent flows, and the impact of its variation on the dynamics should be addressed in detail. Most studies reported in literature have focused on the impact of variable viscosity on the large scales. Chhabra et al. (2005) and Talbot et al. (2013) studied turbulent jet flows, where the viscosities of the jet and host fluids differ. They observed, compared to CV flow, that the relationship between production and dissipation of turbulent energy is altered and that the entrainment and the spreading rates of the

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jet flow are changed. An altered spreading rate was also observed by direct numerical simulations of turbulent shear layers with variable viscosity by Taguelmimt et al. (2016a,b) indicating the ability of viscosity variations to modify the largest scales of the flow. Voivenel et al. (2017) further confirmed these findings using laser measurements and derived generalized scale-by-scale budget equations for velocity increments in inhomogeneous and anisotropic turbulence with variable viscosity. Based on this work, Danaila et al. (2017) showed that variable viscosity effects can invalidate the self-similarity of turbulent jet flows. The analysis of Lee et al. (2008) focused on the turbulent mixing of two initially segregated fluids with different viscosity in homogeneous isotropic turbulence. They found that the dissipation becomes independent of viscosity confirming Taylor's postulate. Taylor (1935) postulated that the mean energy dissipation $\langle\varepsilon\rangle$ depends only on the large-scale velocity fluctuations u' and the integral length scale l_t, i.e.

$$\langle\varepsilon\rangle \propto \frac{u'^3}{l_t}, \tag{1}$$

and hence becomes independent of viscosity, provided that the Reynolds number is sufficiently large. In Eq. (1), the characteristic large-scale velocity fluctuations are defined as

$$u' = \sqrt{\langle u_i^2\rangle/3}, \tag{2}$$

and the integral length scale l_t is defined as

$$l_t = \frac{3\pi}{4}\frac{\int \kappa^{-1}E(\kappa,t)\mathrm{d}\kappa}{\int E(\kappa,t)\mathrm{d}\kappa}, \tag{3}$$

with $E(\kappa)$ being the three-dimensional energy spectrum and κ the magnitude of the wave-number vector. Ensemble-averages are denoted by angular brackets and Einstein's summation convention is used, which implies summation over indices appearing twice.

In this paper, we investigate the decay of homogeneous isotropic turbulence in a variable viscosity fluid by means of highly resolved direct numerical simulations (DNS). We address the question how quantities such as the kinetic energy and the associated dissipation rate, as well as the inter-scale transport mechanism of turbulence are changed by local fluctuations of the viscosity. The paper is structured as follows. Section 2 presents the governing equations and the direct numerical simulations on which the analysis is based. Section 3 introduces the one-point budget equation of the turbulent kinetic energy and discusses the impact of variable viscosity on the dissipation mechanism of turbulence. Section 4 addresses the impact of variable viscosity on the viscous cut-off scales of turbulence. An analysis of the inter-scale transport mechanism based on a budget equation for the second-order velocity structure function is presented in Section 5. We summarize this study in Section 6.

2. Direct numerical simulations and governing equations

Direct numerical simulations of homogeneous isotropic turbulence with variable viscosity at three different viscosity ratios, between 1 and 15 have been performed. The DNS solves the three-dimensional incompressible Navier–Stokes equations,

$$\frac{\partial u_j}{\partial t} + u_i\frac{\partial u_j}{\partial x_i} = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_i}(2\nu s_{ij}), \tag{4}$$

with the continuity equation

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{5}$$

in a triply periodic box with size 2π by a pseudo-spectral method. In eqs. (4) and (5), the velocity field is denoted by u_j, p is the pressure (for simplicity the density $1/\rho$ is incorporated in p), ν is the local viscosity, and s_{ij} is the strain-rate tensor, defined as

$$s_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right). \tag{6}$$

Note that because the viscosity fluctuates, Eq. (4) contains one supplementary term ($\partial\nu/\partial x_i s_{ij}$), which introduces another non-linearity (ν depends on the scalar, which is transported by the velocity field).

The local viscosity field $\nu(\boldsymbol{x}, t)$ is determined by solving an advection-diffusion equation for a scalar field $\phi(\boldsymbol{x}, t)$, i.e.

$$\frac{\partial\phi}{\partial t} + u_i\frac{\partial\phi}{\partial x_i} = D\frac{\partial^2\phi}{\partial x_i^2}. \tag{7}$$

The scalar ϕ is statistically isotropic and homogeneous. For numerical convenience the scalar is further bounded, i.e. $-1 \le \phi(\boldsymbol{x}, t) \le 1$, and has zero mean, i.e. $\langle\phi\rangle = 0$. Ensemble-averages are computed due to homogeneity over the full computational domain. The local viscosity $\nu(\boldsymbol{x}, t)$ is linked through a linear relation to the scalar field, e.g. Gréa et al. (2014)

$$\nu(\boldsymbol{x}, t) = \langle\nu\rangle + \nu'(\boldsymbol{x}, t) = \langle\nu\rangle + c\phi(\boldsymbol{x}, t), \tag{8}$$

where $\langle\nu\rangle$ denotes the uniform mean viscosity, $\nu'(\boldsymbol{x}, t)$ denotes the fluctuating viscosity field, and c is a positive constant with $c < \langle\nu\rangle$ to ensure positivity of $\nu(\boldsymbol{x}, t)$. A linear relation between ν and ϕ is convenient because it keeps the mean viscosity $\langle\nu\rangle$ unchanged during the decay. The constant c is obtained from the initial minimum and maximum values of the viscosity by $c = (\nu_{\max} - \nu_{\min})/2$, which implies that the initial scalar variance $\langle\phi^2\rangle$ equals unity. In the following, we use ϕ as a surrogate for ν. The molecular diffusivity D in Eq. (7) is assumed to be constant and equals the mean viscosity $\langle\nu\rangle = (\nu_{\min} + \nu_{\max})/2$. As a consequence, the Schmidt number, defined as $Sc = \nu/D$, is a fluctuating quantity.

The following paragraph briefly summarizes the main features of the DNS. More details about the numerical procedure and the parallelization approach are given by Gauding et al. (2015, 2017). Adapting the approach by Mansour and Wray (1994), the Navier–Stokes equations are formulated in spectral space as

$$\frac{\partial}{\partial t}(\hat{u}_i\exp(\nu\kappa^2 t)) = \exp(\nu\kappa^2 t)P_{ij}\widehat{H}_j, \tag{9}$$

where

$$\widehat{H}_j = -i\kappa_i\mathscr{F}(u_iu_j - 2c\phi(\boldsymbol{x}, t)s_{ij}) \tag{10}$$

is the Fourier transform of the non-linear terms, including the convective term and the non-linear part of the viscous term. The wave-number vector is denoted by $\boldsymbol{\kappa}$, and the Fourier transform of the velocity field is denoted by $\hat{\boldsymbol{u}}$. The projection operator $P_{ij} = \delta_{ij} - \kappa_i\kappa_j/\kappa^2$ imposes incompressibility. The non-linear terms are computed in physical space and a truncation technique with a smooth spectral filter is applied to remove aliasing errors. The smooth spectral filter is highly localized in both real and spectral space, and was found to be dynamically very stable, cf. Hou and Li (2007). This feature is relevant for the present study to prevent instabilities at high wave-number modes caused by the non-linear part of the viscous term. An integrating factor technique is used for an exact integration of the linear part of the viscous terms. Temporal integration is performed by a low-storage, stability preserving, third-order Runge–Kutta scheme. An additional necessary constraint that has to be satisfied by the DNS is an adequate resolution of the smallest scales. As proposed by Mansour and Wray (1994), we require that for all times, the condition $\kappa_{\max}\eta_0 \ge 1$ is satisfied, where $\eta_0 = (\langle\nu\rangle^3/\langle\varepsilon\rangle)^{1/4}$ is the Kolmogorov length scale and $\kappa_{\max}$ is the largest resolved wave-number. A grid resolution of 1024^3 points is used to appropriately account for both small and large scales.

Let us now turn our attention to the initialization of the DNS. For the velocity field, we follow the approach of Ishida et al. (2006) and prescribe a broad-band energy spectrum of the type

$$E(\kappa, t=0) \propto \kappa^4\exp(-2(\kappa/\kappa_p)^2). \tag{11}$$