International Journal of Heat and Fluid Flow 71 (2018) 200–209

Contents lists available at ScienceDirect

International Journal of Heat and Fluid Flow

journal homepage: www.elsevier.com/locate/ijhff

A modified Parametric Forcing Approach for modelling of roughness

Pourya Forooghi*[,a], Bettina Frohnapfel[a], Franco Magagnato[a], Angela Busse[b]

[a] Institute of Fluid Mechanics, Karlsruhe Institute of Technology, Kaiserstraße 10, Karlsruhe 76149, Germany
[b] School of Engineering, University of Glasgow, Glasgow G12 8QQ, UK

ARTICLE INFO

Keywords:
Roughness modelling
Parametric forcing
Turbulent channel flow

ABSTRACT

Surface roughness in turbulent channel flow is effectively modelled using a modified version of the Parametric Forcing Approach introduced by Busse and Sandham (2012). In this modified approach, the model functions are determined based on the surface geometry and two model constants, whose value can be fine tuned. In addition to a quadratic forcing term, accounting for the effect of form drag due to roughness, a linear forcing term, analogous to the Darcy term in the context of porous media, is employed in order to represent the viscous drag. Comparison of the results with full-geometry resolved Direct Numerical Simulation (DNS) data for the case of dense roughness (frontal solidity ≅0.4) shows a satisfactory prediction of mean velocity profile, and hence the friction factor, by the model. The model is found to be able to reproduce the trends of friction factor with morphological properties of surface such as skewness of the surface height probability density function and coefficient of variation of the peak heights.

1. Introduction

Study of turbulent flows over rough surfaces finds application in several engineering – e.g. turbomachinery, marine transportation and ice accretion on aircrafts – and geophysical – e.g. wind flow over plant and urban canopies – problems. Roughness causes an increase in the friction factor.

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U_b^2} \tag{1}$$

In Eq. (1) τ_w and U_b denote wall shear stress and bulk velocity. It is also well established that roughness leads to a shift ΔU^+ in the logarithmic law of the wall (Nikuradse, 1933; Hama, 1954).

$$U^+(y) = \frac{1}{\kappa} ln(y^+) + 5.5 - \Delta U^+ \tag{2}$$

where $\kappa = 0.4$ is the von Kármán constant and the value 5.5 is the log-law intercept for a smooth wall. It can be shown that an increase in the roughness function ΔU^+ corresponds to an increase in friction factor (Jimenez, 2004; Flack and Schultz, 2010).

Alternatively, Eq. (2) can be written as

$$U^+(y) = \frac{1}{\kappa} ln\left(\frac{y}{k_s}\right) + 8.5 \tag{3}$$

where roughness function ΔU^+ is replaced by the interchangeably usable quantity k_s – effective or equivalent sand-grain roughness height (Jimenez, 2004). For a majority of practical rough surfaces (so called k-type roughness), k_s is "proportional to the dimensions of roughness elements", provided that the roughness elements are large enough to fall into the 'fully-rough' regime (Jimenez, 2004). The ratio of k_s to the physical characteristic dimension k of roughness is a function of the surface geometry (Jimenez, 2004). A comprehensive review on the dependence of the ratio k_s/k on different geometrical surface parameters has been undertaken by Flack and Schultz (2010). Recently, Forooghi et al. (2017) and Thakkar et al. (2017) investigated several irregular rough surfaces using DNS in order to determine the most important surface parameters for the prediction of flow properties, i.e. ΔU^+ or k_s. There is a consensus among above references that, at constant roughness density, flow properties are most sensitive to the skewness Sk of the surface height probability distribution function. Surface slope also plays an important role in determining both skin friction and physics of the flow. With a decrease in effective slope – defined as mean absolute streamwise surface slope – form drag loses its dominance in the momentum exchange between the surface and flow (Napoli et al., 2008; Schultz and Flack, 2009).

DNS in which the details of surface geometry are resolved is required to guarantee that both roughness and flow scales are properly accounted for. A number of such simulations have been published in the past, in which the surface geometry is captured either by body conforming grids (Choi et al., 1993; Chan et al., 2015) or immersed boundary method (IBM) (Orlandi and Leonardi, 2006; Bhaganagar, 2008; Busse et al., 2015; Forooghi et al., 2017; Mazzuoli and Uhlmann,

* Corresponding author.
E-mail address: forooghi@kit.edu (P. Forooghi).

https://doi.org/10.1016/j.ijheatfluidflow.2018.03.019
Received 8 November 2017; Received in revised form 29 March 2018; Accepted 29 March 2018

2017). Both approaches are extremely demanding in terms of computational cost and/or grid generation effort. A way to avoid such difficulties is using a modified version of the Navier–Stokes equation near the wall, in which roughness is 'effectively' modelled. These models, clearly, do not process the degree of fidelity that full-surface resolved DNS provides, thus, require careful verification. In the framework of Reynolds-averaged Navier–Stokes, for instance, so-called Discrete Element Method (DEM) has been used for a long time (Taylor et al., 1985; Tarada, 1990). In DEM, roughness geometry is represented by simple roughness elements and the mass and momentum conservation equations are averaged over control volumes containing several of these elements. Effects of form drag and vortex shedding from roughness elements enter the momentum and turbulent kinetic energy equations through source terms; consequently, not only the momentum equation but also turbulence transport equations contain extra 'modelled' terms.

The idea of modifications in Navier–Stokes equation for roughness modelling has also been used in LES and DNS context. Cui et al. (2003) suggested an approach in which an arbitrary rough surface is decomposed into two parts: resolved scale and sub-grid scale roughness, for the former immersed boundary method and for the latter a random body-force model is used. For very high Reynolds numbers where the roughness height falls below the first near-wall grid point, Anderson and Meneveau (2010) suggested an LES model in which a body force is applied within the first grid-point. The value of the body force is determined based on total incoming momentum flux into the roughness.

Busse and Sandham (2012) proposed a Parametric Forcing Approach (PFA) in which the effect of roughness is introduced by adding the body force term $-\alpha_i F_i(y) u_i |u_i|$ to the otherwise-unchanged Navier–Stokes equation (no summation over index i). u_i denotes instantaneous velocity and $i = 1, 2, 3$ indicate streamwise, wall-normal and spanwise directions corresponding to x, y, z coordinates, respectively. α_i and $F_i(y)$ are referred to as 'roughness factor' and 'roughness shape function' by these authors, respectively. They further simplify the model by applying $\alpha_1 F_1 = \alpha_3 F_3 = \alpha F$ and $\alpha_2 = 0$. By using a DNS grid, PFA involves no other modelling except for the forcing term that represents the momentum exchange between the flow and roughness, therefore, it is possible to purely evaluate the performance of roughness modelling terms. In the PFA introduced in Busse and Sandham (2012) the function αF is not directly related to a specific roughness geometry, therefore, cannot be determined a priori.

The present work aims at a modified version of PFA, in which – apart from the tunable scalar model constants – the forcing amplitude can be determined a priori for a desired roughness geometry, so that the mean flow profile and, thus, the 'friction factor' can be predicted correctly. The model is expected to satisfactorily capture the trends of friction factor with two important topographical surface parameters, i.e. 'skewness' and 'coefficient of variation of roughness peak heights'. Full-geometry resolved DNS data from Forooghi et al. (2017) is used to evaluate the model and its capability to follow the physical trends.

2. Roughness samples

Four roughness samples with systematically chosen geometrical surface parameters are considered in the present paper. The full-geometry resolved DNS for these surfaces have been reported by Forooghi et al. (2017); in the present work the 'geometrical functions' required in the modified PFA (details in Section 3) are calculated for the same samples and the results are compared. The geometry of roughness is generated using an algorithm explained in full in Forooghi et al. (2017), which creates 3D irregular rough surfaces $\tilde{k}(x, z)$. Briefly, the geometry is generated by mounting axisymmetric roughness elements with prescribed shape and spacing in a random pattern on a smooth 'reference plane' which is the lower boundary of the computational domain. Certain topographical properties of the roughness can be adjusted in this approach. Before discussing these properties, it should be stressed that in the present study we focus on the roughness elements with high slopes. As discussed in the introduction, a rough surface with low slope does not behave in the same way as 'normal' roughness does. Schultz and Flack (2009) showed that when the surface slope falls below a certain threshold, the effective roughness height does not scale with the physical dimensions of roughness; therefore, they proposed calling this type of surfaces 'wavy' instead of 'rough'. These authors also suggested a threshold of 0.35 for the effective slope of a wavy surface. Yuan and Piomelli (2014), later on, found a considerably higher threshold equal to 0.7. The surfaces used in this work all have an effective slope equal to 0.88 which should be high enough to avoid any 'waviness' behaviour.

Table 1
Summary of surface samples used in the present study and the values of Reynolds number in the simulations. Results for cases *Ia, II* and *III* are discussed in details in Forooghi et al. (2017).

Sample	k/H	k_{MD}/H	Sk	Δ	k^+	Re_τ
Ia	0.12	0.074	0.21	0.7	67	498
Ib	0.06	0.037	0.21	0.7	32	500
II	0.12	0.047	0.66	0.7	64	502
III	0.19	0.1	0.21	0	110	499

As discussed in the introduction, data published in the literature suggest that, at a constant effective slope, skewness Sk defined as

$$Sk = \frac{1}{A \cdot k_{rms}^3} \int_A (\tilde{k} - k_{MD})^3 dA, \quad k_{rms}^2 = \frac{1}{A} \int_A (\tilde{k} - k_{MD})^2 dA \tag{4}$$

can control the effective roughness height to a high extent. In Eq. (4), A is the surface area projected on the reference plane, shortly wall-projected area, and k_{MD} (melt-down height) is the mean surface height. $\tilde{k}$, which is the surface height from the reference plane, is a function of coordinates in y-normal plane, i.e. $\tilde{k}(x, z)$. Forooghi et al. (2017) found that at constant skewness, a roughness composed of 'uniform' elements shows a higher resistance to flow than one with non-uniform elements. To measure the non-uniformity of the peak heights a 'coefficient of variation' Δ, defined as the height difference between the highest and the lowest peaks of the surface normalized with the mean peak height, is used.

Table 1 summarizes the geometrical properties of the surface samples. Sample *Ia* is used as the control case. Compared to this sample, sample *II* has a higher skewness but a similar Δ, while sample *III* has a similar Sk but its Δ is zero (uniform peak heights). Sample *Ib* has the same topographical properties as *Ia* but its dimensions are scaled down. Mean roughness peak height k[1] is halved in *Ib* compared to *Ia*. The values of k^+ shown in the table suggest that the studied surfaces likely span all the way between the transitionally-rough and fully-rough regimes, which facilitates assessing the versatility of the model under investigation.

The values of friction velocity Re_τ for each case – similar for the reference DNS and present simulations – are also listed in Table 1.

$$Re_\tau = \frac{u_\tau (H - k_{MD})}{\nu} \tag{5}$$

In Eq. (5), H is the distance between the bottom plane and the middle of the channel, i.e. the wall-normal dimension of the computational domain (see Section 4 for the complete description of the computational set-up), therefore, the length scale $(H - k_{MD})$ used in the definition of Reynolds number is the half-height of a channel with the same cross-section area or, namely, the 'effective' half-height of the channel. The friction velocity $u_\tau = (\tau_w/\rho)^{\frac{1}{2}}$ is based on the wall shear stress and is calculated from the integral momentum balance using the mean

[1] This quantity is used as the representative dimension of the roughness throughout the paper.