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Forced convection heat transfer of Giesekus fluid with wall slip above the critical shear stress in pipes

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ABSTRACT

Forced convective heat transfer in pipes is investigated for viscoelastic fluids obeying the Giesekus constitutive equation including effect of slip condition by an approximated analytical method. The slip equation at wall is considered as a nonlinear Navier model with non-zero slip critical shear stress. The problem under consideration is steady, laminar and fully developed. Thermal boundary conditions are assumed peripherally and axially constant heat flux at wall. The fluid heating and cooling cases are considered for analysis. Dimensionless temperature profiles and Nusselt number are obtained by solving governing equations and the effects of slip parameters, viscous dissipation and fluid elasticity are discussed. Results show that Nusselt number increases by increasing slip effect but decreases by increasing Brinkman number for the case of fluid heating. However, for the cooling case, the heat generated by viscous dissipation can overcome the effect of wall cooling at first critical Brinkman number and fluid starts to warm up. Also the Nusselt curve shows a singularity in a second critical Brinkman number.

1. Introduction

The heating and cooling processes have to use the non-Newtonian fluids in a broad variety of equipment and industries related to fluid such as shell and tube heat exchangers, polymer and plastic extrusion, drilling operations and food industries (Chhabra and Richardson, 1999; Yamaguchi, 2008). Therefore, the knowledge of heat transfer is mandatory for the equipment design and quality control of the final products. Also the empirical evidences indicate that, in certain circumstances, most of these complex fluids may be slipped at the solid boundary which again has a strong influence to quality of final products such as sharkskin, stick-slip, and gross melt fracture instabilities in polymer extrusion. Slip can be occurred by three mechanisms as below:

- Adhesive failure of the polymer chains on the solid surface leading to detachment of the absorbed chains from the wall.
- Cohesive failure arising from disentanglement of the bulk chains from chains adsorbed at the wall, and then disentangled chains will slip over adsorbed chains.
- Formation of a low viscosity layer of solvent which is known to low-viscosity mesophase and the bulk polymer chains slip on this layer.

One of the most common models for determine the slip velocity at the wall is the nonlinear Navier slip law which is based on experimental results and consists in a power law relationship between slip velocity and shear stress at the wall as follows:

$$u_w = \left(\frac{|\tau_w|}{\beta}\right)^{\frac{1}{s}} \tag{1}$$

where u_w, τ_w and s are the slip velocity, shear stress at wall and power law index respectively.

β is the slip coefficient which depends on temperature, normal stress, molecular parameters and properties of the fluid/wall interface (Denn, 2001). As $\beta \to 0$ full slip flow and $\beta \to \infty$ no slip boundary condition are recovered. Fig. 1 shows the Hagen–Poiseuille velocity profile for slip and no slip boundary conditions.

Since the empirical evidence shows the slip occurs only when wall shear stress exceeds a critical value (Mohseni and Rashidi, 2015) therefore the nonlinear Navier slip model is employed under the following form:

$$\begin{cases} u_w = 0 & |\tau_{rz\,w}| \le \tau_c \\ u_w = \left(\frac{|\tau_{rz\,w}| - \tau_c}{\beta}\right)^{\frac{1}{s}} & |\tau_{rz\,w}| > \tau_c \end{cases} \tag{2}$$

Effect of slip condition in the flow field of Newtonian and non-Newtonian fluids has been investigated extensively (Damianou et al.,

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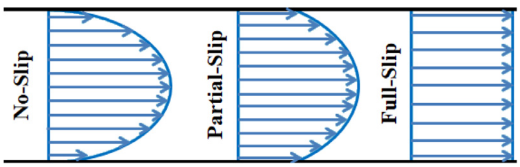

Fig. 1. Schematic diagram of Poiseuille flow with slip boundary conditions.

2014; Ferras et al., 2012a, 2012b; Matthews and Hill, 2007; Chatzimina et al., 2009; Kalyon and Malik, 2012; Pereira, 2009; Kaoullas and Georgiou, 2013; Housiadas, 2013; Damianou et al., 2013; Tang and Kalyon, 2008a, 2008b) but research on heat transfer is scant. Analytical solutions for heat transfer and entropy generation of Newtonian fluid in microchannel were obtained by Anand (2014) considering slip boundary conditions. The non-linear Navier, Hatzikiriakos and asymptotic slip laws were employed and the microchannel walls were subject to uniform heat flux. Finally, the effect of slip at walls on velocity distribution, temperature distribution, Nusselt number, entropy generation rate and Bejan number has been reported in this paper. Shojaeian and Kosar (Shojaeian and Kosar, 2014) investigated effect of slip condition on convective heat transfer and entropy generation for Newtonian and non-Newtonian fluid using the linear Navier slip model between parallel-plates. The thermal boundary conditions were assumed isoflux and isothermal and the expressions for velocity, local and mean temperature distributions, Nusselt number, entropy generation and Bejan number were obtained analytically. The slip effect on flow and thermal fields of Ostwald–de Waele power law fluid in circular microchannel are studied by Barkhordari and Etemad (2007) using control volume finite difference method. The slip velocity in their study is defined as a constant coefficient of mean velocity of fluid and thermal boundary conditions are considered constant temperature and constant heat flux at wall. Eventually, the influence of slip coefficient on friction factor and Nusselt number were investigated. Mahjoob et al., (2009) performed a similar research in rectangular microchannel. To the best of our knowledge, the slip effect on convective heat transfer of viscoelastic fluid, with effect of critical shear stress in slip model, has not been yet investigated. Then, we propose in the present study an analytical approach of forced convection heat transfer in pipe for laminar, steady state and fully developed flow of nonlinear viscoelastic fluid obeying Giesekus model with accounting slip effect. The slip law at wall employed is the nonlinear Navier one with non-zero slip critical shear stress. The canonical geometry of pipe flow is considered in the present paper regarding its wide range of applications.

2. Governing equation

The problem under consideration is steady, laminar, thermally and hydrodynamically fully developed flow in a pipe (see Fig. 2). Axial heat conduction is neglected compared to the radial heat transfer by the order of magnitude analysis (Kakac and Yener, 1995). The effect of viscous dissipation is included due to the high viscosity of viscoelastic fluids considered. Thermophysical properties of fluid are taken

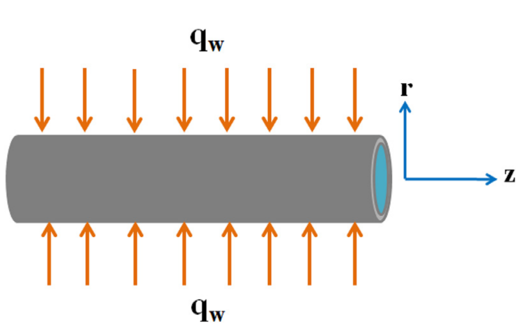

Fig. 2. Schematic diagram of the pipe and its thermal boundary conditions.

independent of temperature. This assumption sounds reasonable since temperature variations are not high enough to significantly change fluid properties (Oliveira and Pinho, 2000; Bird et al., 1987; Mohseni et al., 2015).

The continuity, momentum and Giesekus constitutive equations (without retardation time) are:

$$\nabla u = 0 \tag{3a}$$

$$\rho \frac{Du}{Dt} + \nabla P = \nabla \tau \tag{3b}$$

$$\tau + \frac{\alpha\lambda}{\eta}(\tau \cdot \tau) + \lambda \frac{\vartheta \tau}{\vartheta t} = \eta \dot{\gamma} \tag{3c}$$

where

$$\dot{\gamma} = [\nabla \mathrm{u} + (\nabla \mathrm{u})^T] \tag{4}$$

$$\frac{\vartheta \tau}{\vartheta t} = \frac{D\tau}{Dt} - [\tau \cdot \nabla \mathrm{u} + (\nabla \mathrm{u})^T \tau] \tag{5}$$

$$\frac{D\tau}{Dt} = \frac{\partial \tau}{\partial t} + (\mathrm{u}\nabla)\tau \tag{6}$$

η and λ are the model parameters representing zero shear viscosity and zero shear relaxation time, respectively (Giesekus, 1983). In particular, the zero shear relaxation time is corresponding to the time that the stresses arising from shear rate relax after the fluid motion has stopped, which is characteristic of viscoelastic fluids (Bird et al., 2001). The model parameters are function of shear rate. They approach to a constant value at very low shear rate which are named zero shear model parameters. Parameter α in Eq. (3c), lying in the range $0 \le \alpha \le 1$ (Giesekus, 1982) is a mobility factor. The term containing α in the constitutive equation is attributed to anisotropic Brownian motion and/or anisotropic hydrodynamic drag on the constituent polymer molecules (Bird et al., 1987).

Dimensionless quantities are as follows:

$$r^* = \frac{r}{R} \quad z^* = \frac{z}{R} \quad u^* = \frac{u_z}{U} \quad \gamma^{\circ *} = \frac{\gamma^{\circ}}{U/R} \quad \tau^* = \frac{\tau}{\eta U/R} \quad \psi = \frac{R^2}{\eta U}\left(\frac{dp}{dz}\right)$$

Where U is the average velocity over cross-section of the pipe and described as follows:

$$U = \frac{\int_0^R 2\pi\, r\, u_z\, dr}{\int_0^R 2\pi\, r\, dr} \tag{7}$$

3. Analytical solution

3.1. Hydrodynamic solution

The shear stress equation is derived from Eq. (3b) as follows:

$$\tau_{rz}^* = \frac{\psi\, r}{2} \tag{8}$$

The shear rate equation was derived from Giesekus equation by Yoo and Choi (1989) as follows:

$$\gamma_{rz}^* = \frac{\mathrm{du}^*}{\mathrm{dr}^*} = 2\alpha\tau_{rz}^* \frac{1 \pm (2\alpha - 1)\sqrt{1 - 4\alpha^2 De^2 \tau_{rz}^{*2}}}{\left(2\alpha - 1 \pm \sqrt{1 - 4\alpha^2 De^2 \tau_{rz}^{*2}}\right)^2} \tag{9}$$

De is the Deborah number, defined as ($De = \lambda U/R$), which is related to the level of fluid elasticity.

The dimensionless form of the slip boundary condition is as follows:

$$\begin{cases} \mathrm{u}_w^* = 0 & |\tau_{rzw}^*| \le B_C \\ \mathrm{u}_w^* = \left(\frac{|\tau_{rzw}^*| - B_C}{B}\right)^{\frac{1}{s}} & |\tau_{rzw}^*| > B_C \end{cases} \tag{10}$$

B and B_c are dimensionless slip number and dimensionless slip