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Relation between a singly-periodic roughness geometry and spatio-temporal turbulence characteristics

J. Morgan*, B.J. McKeon

Graduate Aerospace Laboratories, California Institute of Technology, Pasadena, CA 91125, United States

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ABSTRACT

The structure of a turbulent boundary layer over a singly-periodic roughness of large wavelength is shown to give insight into the physics of rough-wall boundary layers. To this end, a roughness consisting of a single spanwise-varying mode and a single streamwise-varying mode was 3D printed with wavelengths on the order of the boundary layer thickness. The large length scale introduced by such a roughness creates spatial inhomogeneity of the mean velocity field throughout the entire boundary layer. A hot-wire probe was used to take time series of streamwise velocity at a grid of points in the x,y, and z directions, covering the volume over a single period of roughness, and allowing Fourier transforms of field variables to isolate the spatial variations correlating to the periodic geometry.

The pre-multiplied Taylor-transformed wavelength power spectrum of streamwise velocity $\lambda_T\Phi(y, \lambda_T, x, z)$ can be Fourier-transformed in space to reveal that the portion of the power spectrum which varies most strongly in the streamwise direction is the portion with Taylor-transformed wavelength λ_T equal to the roughness wavelength λ_x. The spatial variation of the power spectrum at this wavelength exhibits a systematic change in phase across the boundary layer, which can be correlated to the phase of the spatially-varying time-averaged velocity field to reveal amplitude modulation of particular wavelengths by a roughness-induced synthetic scale.

In a canonical smooth-wall boundary layer, the spatial variation of the mean velocity and the power spectrum would be identically zero due to translational symmetry. The introduction of a periodic roughness introduces the spatial variation in the power spectrum, but not directly. The roughness creates a stationary time-averaged velocity mode, but this mode does not appear in the power spectrum as it does not convect. The connection to the power spectrum must therefore be through non-linear interactions. It is shown that the correlation between the mean velocity and the power spectrum can be interpreted exactly as a measure of phase organization between pairs of convecting velocity modes which are triadically consistent with the stationary roughness velocity mode, analogously to amplitude modulation in canonical flows. Implications for real-world roughness are discussed.

1. Introduction

1.1. Similarity in rough-wall turbulent boundary layers

The canonical fully-developed smooth-wall boundary layer can be characterized by a single parameter, namely a Reynolds number formed from the freestream velocity U_∞, streamwise coordinate x, and kinematic viscosity ν, $Re = U_\infty x/\nu$, or alternatively the friction Reynolds number, $Re_\tau = u_\tau\delta/\nu$, where the friction velocity $u_\tau = \sqrt{\tau_w/\rho}$ is given by the square root of the wall shear stress divided by the density and δ is the boundary layer thickness. A rough-wall boundary layer complicates that characterization by adding length scales corresponding to the geometry of the roughness.

Although each individual roughness pattern presents a unique physical and mathematical case, some trends emerge across a wide variety of geometries. Flows without sufficient scale separation between the roughness height k and the boundary layer thickness δ are characterized as "obstacle flows" and have significant qualitative differences in measured flow quantities throughout the boundary layer when compared to smooth wall flows (Jimenez, 2004). Jimenez gives a criterion of $\delta/k < 40$ to separate obstacle flows from rough-wall flows, which do exhibit some similarity to canonical smooth-wall flows. Our focus here is restricted to rough-walls.

Wall flows with three-dimensional roughness and sufficient scale separation δ/k exhibit similarity with smooth-wall flows beyond just the logarithmic mean profile. Townsend's hypothesis (Townsend, 1980)

* Corresponding author.
E-mail addresses: jmorgan@caltech.edu (J. Morgan), mckeon@caltech.edu (B.J. McKeon).

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posits that, with sufficient separation between the roughness and the outer scales of the flow, the boundary layer physics outside of the roughness sublayer are affected by the roughness only through the length scales and velocity boundary conditions which the sublayer imposes on the rest of the flow. In this description, the roughness elements serve only to perturb the turbulent cascade in the roughness sublayer, altering the velocity profile near the wall and therefore the wall shear stress. The resulting values of u_τ and δ then provide the scales for the flow statistics in the outer layer, so that rough-wall quantities of the form $Q^+(y/\delta)$ are identical to smooth-wall quantities at the same Reynolds number, where a superscript plus denotes a quantity normalized by “inner units” ν and u_τ. Schultz and Flack (2007) found such a similarity for the velocity defect $U_\infty^+ - U^+$ and for single-point velocity correlations up to third order in a turbulent boundary layer for a three-dimensional roughness created by oblique bi-directional sanding. Flores and Jimenez (2006) also provide evidence for this view, observing roughness-like similarity for simulated channel flows in which wall roughness was replaced with prescribed velocities and Reynolds stresses at the wall.

In contrast, a parametric study of heterogeneous, herringbone-pattern riblet roughness by Nugroho et al. (2013) found that for such a periodic roughness, large spatial variations in mean flow quantities can persist throughout the boundary layer, with boundary layer thickness varying by a factor of two within a single spanwise period of roughness. Mejia-Alvarez and Christensen (2013) discovered large, δ-scale variations in ensemble-averaged flow velocity within the roughness sublayer even in a disordered, real world roughness derived from a damaged turbine blade. Further studies by Barros and Christensen (2014) and Anderson et al. (2015) correlated areas of recessed roughness to low-momentum pathways and elevated roughness to high-momentum pathways. They further found associated secondary flows which persisted well into the outer layer of the boundary layer. Similar to the work of Hinze (1967) on flows in the corners of rectangular ducts, these flows were found to be Prandtl’s secondary flow of the second kind, generated by spanwise gradients in Reynolds stress. Studies of streamwise-aligned heterogeneous roughness by Vanderwel and Ganapathisubramani (2015) and Willingham et al. (2014) found that these secondary flows can extend through the boundary layer with a scale on the order of δ when the spanwise spacing is appropriately large. Spanwise-aligned roughnesses, including both two-dimensional bars and staggered cubes, were found by Volino et al. (2011) to effect the flow well into the outer region via blockage effects. These studies on heterogeneous roughness indicate a number of circumstances under which a roughness will not obey Townsend’s hypothesis, particularly when the roughness is coherent in the streamwise and spanwise directions with large length scales.

Mejia-Alvarez and Christensen (2010) explored the effects of individual roughness scales by using proper orthogonal decomposition to extract a low-order representation of a real-world roughness. A 3D-printed low-order roughness constructed from the fifteen most amplified proper orthogonal decomposition (POD) modes of the surface height variation was found to accurately reproduce the drag characteristics of the full roughness in turbulent boundary layer flow, indicating that a key subset of geometric scales are responsible for the flow physics of real-world rough-wall flows. The present work proceeds in the opposite sense, by creating a simple, singly-periodic roughness to observe the effect on the flow of a single large roughness scale.

1.2. Amplitude modulation of small-scale turbulence by large-scale structures

Large scale velocity disturbances are known to be correlated with small-scale flow physics in canonical smooth-wall flows. A deterministic coupling between large- and small-scale structures in shear flows was first observed by Rao et al. (1971). Mathis et al. (2009) went on to quantify the correlation with the amplitude modulation correlation coefficient R. Under this approach, the large scale velocity fluctuations u_L and small scale fluctuations u_S are separated from the full velocity time series by a frequency filter and considered as independent signals. The envelope of the small scale fluctuations E is calculated as a function of time using the Hilbert transform. The envelope is then filtered to isolate large-scale modulation of the envelope, resulting in the time series E_L. This quantity is then compared to the large scale fluctuations using the temporal correlation coefficient to yield the amplitude-modulation correlation coefficient, R, where an overbar indicates a time average:

$$R = \overline{u_L E_L} / \left(\sqrt{\overline{u_L^2}} \sqrt{\overline{E_L^2}} \right) \tag{1}$$

In smooth-wall flows, R attains a maximum in the viscous region, approaches then passes through zero in the log region, attains a minimum in the wake region before increasing in the intermittent turbulent/non-turbulent region at the edge of the boundary layer.

Jacobi and McKeon (2013) demonstrated that R is dominated by the signature of one scale, associated with the very large-scale motions in the flow. For periodic signals, such a correlation coefficient can be cleanly interpreted as the relative phase between signals via the dot product, per Chung and McKeon (2010). Duvvuri and McKeon (2015) showed R to be a measure of average phase for pairs of turbulent scales which are triadically consistent with the large scales. Furthermore, those authors probed the phase organization between scales by perturbing a boundary layer with an oscillating transverse rib, introducing a strong synthetic large-scale mode into the flow. A new correlation coefficient, analogous to R above but associated with just the synthetic component of the large-scale signal, was defined as

$$\Psi = \overline{\tilde{u}\widetilde{u_S^2}} / \left(\sqrt{\overline{\tilde{u}^2}} \sqrt{\overline{\widetilde{u_S^2}^2}} \right) \tag{2}$$

Here tildes refer to a phase average with a period equal to the oscillation period of the rib.

For the two-dimensional, spanwise constant disturbance, the quantity Ψ was found to be near one close to the wall, indicating perfect correlation. Around the critical layer of the flow (the wall-normal location at which the mean velocity is equal to the convection speed of the synthetic mode), Ψ changed abruptly to nearly -1 for nearly a decade of height, indicating perfect anti-correlation. Ψ then increased toward 1 at the edge of the boundary layer. In this way, it was shown that a synthetic mode organizes the phases of triadically-consistent scales in a quasi-deterministic manner.

1.3. Objectives

The present work aims to explore the relationship between roughness geometry and boundary layer physics with an idealized sinusoidal roughness. The sinusoidal roughness alters the boundary condition of the flow in a simply-identifiable way, which creates a static, inhomogeneous temporal-mean velocity field. This inhomogeneity reaches far from the wall, and can be characterized by a simple spatial spectral composition. Due to the large wall-parallel wavelengths compared to the boundary layer thickness and non-negligible amplitude, the effects of the roughness extend through much of the boundary layer, and hot wire anemometry can be used to measure the spatial variation in mean quantities, statistics, and power spectra required to trace the effects of the roughness. Due to the nominally-linear nature of the boundary condition, it is proposed that the effects of a number of these simple roughnesses can be linearly superposed to predict the behavior of a real-world roughness in wall-bounded flow.

The paper proceeds as follows. Sections 2 and 3 describe the experimental set-up and analytical approaches employed in this study. Results are reported in Section 4 and discussed in Section 5, before a summary and conclusions in Section 6.