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A finite difference solution for freezing brine on cold substrates of spongy ice

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ABSTRACT

The process of rapid freezing of a thin layer of brine, suddenly in contact with a cold substrate of brine-spongy ice, is investigated. The mechanism of intermittent ice accretion on cold substrates, which occurs in a short period of time, is different from the slow freezing of salt water and must be evaluated using a differential analysis. Investigation of rapid freezing fills a gap of knowledge related to intermittent icing of superstructures, which has usually been studied using control volume methods. The equation of transient heat conduction through brine-spongy ice is developed. Rapid freezing causes complete solute trapping, which makes the salinity constant and stable at the phase interface. A finite difference method, using uniformly-spaced fixed-grid mesh, is employed as a numerical scheme for calculating the rate of ice accretion. A method is presented for discretization at nodes close to the phase interface for preventing the instability of numerical solutions when the phase interface passes the adjacent nodes. The discretization is based on the Method of Lines (MOL) which is a numerical-iterative method of solution. Numerical results show that higher salinities and lower initial temperatures of brine-spongy substrates have the potential to create a thicker layer of new ice. Experimental studies show that the model and numerical solutions accurately predict the rapid freezing of brine on a cold substrate of brine-spongy ice.

1. Introduction

The freezing of a thin layer of salt water and estimating the rate of ice accretion on cold substrates are challenging phenomena related to vessels and offshore structures in cold regions (Dehghani et al., 2018; Horjen, 2013; Kulyakhtin and Tsarau, 2014; Ryerson, 1995; Zakrzewski, 1987). Intermittent ice accretion due to wave-impact sea spray is a dominant mechanism of ice accretion that depends on heat transfer through the spongy substrate (Kulyakhtin and Tsarau, 2014). The brine layer starts freezing by releasing and dissipating the latent heat of fusion through the substrate. The resultant ice is a two-phase medium including pure ice and trapped brine pockets (Blackmore et al., 2002; Makkonen, 1987). This type of freezing involves a two-phase substrate called spongy ice and a thin layer of a solution of water and salt called the brine layer. Trapped brine pockets, existing between pure ice dendrites, fill the porosity of the ice medium and create spongy ice. Accumulation of the first layer of spongy ice on a substrate of spongy ice affects the rate of ice accretion for the next layers (Kulyakhtin et al., 2016; Wettlaufer, 1992a). There are many methods for detecting and measuring ice loads accumulated on structures (Fazelpour et al., 2016, 2017).

Rapid solidification prevents salt rejection into the brine layer during the freezing process. The diffusion rate of the salute is much less than that of the heat which causes the trapping of the rejected salt in new dendrites of ice at the phase interface (Wettlaufer, 1992a, 1992b). The diffusion rate of solute in the brine layer is about 250 times slower than that of the heat into the brine layer (Worster and Wettlaufer, 1997). Therefore, in the case of rapid freezing, the solute will be trapped at the phase interface between the dendrites. As an approximation, for the rate of ice accretion greater than 1.9×10^{-7} mm/s, the unstable mechanism of ice accretion is activated and the process of brine trapping starts (Worster and Wettlaufer, 1997). Other aspects of brine entrapment in sea ice structures, which are different from the rapid freezing, have been investigated (Granskog et al., 2006; Thoms et al., 2013). In addition, general criteria for rapid solidification and solute trapping of some materials have been discussed (Aziz, 1982; Sobolev, 2012; Wettlaufer, 1992a, 1992b; Worster and Wettlaufer, 1997).

Estimating the thickness of spongy ice accumulated on marine structures has been an important subject of research for many years. The method of the control volume, which considers the average of the properties in target systems and is not an accurate method for fast phenomena with high gradients of properties, has been the mainly used method (Horjen, 2013; Kulyakhtin and Tsarau, 2014; Ryerson, 1995; Zakrzewski, 1987). Intermittent ice accretion of wave-impact sea spray (Dehghani et al., 2016, 2017), which sometimes can be considered as a

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Nomenclature

c_b	specific heat capacity of brine (J/kg.K)
c_e	equivalent specific heat capacity of brine-spongy ice (J/kg.K)
c_i	specific heat capacity of ice (J/kg.K)
c_s	specific heat capacity of brine-spongy ice (J/kg.K)
I_s	thickness of accumulated brine-spongy ice (mm)
$I_{s,5‰}$	thickness of accumulated brine-spongy ice with the salinity of 5‰ (mm)
$I_{s,\,-5°C}$	thickness of accumulated brine-spongy ice with the initial temperature of −5°C (mm)
$I_{s,\ final}$	thickness of accumulated brine-spongy ice calculated by the final scheme (mm)
i	node number (—)
k_b	thermal conductivity of brine (W/m.K)
k_i	thermal conductivity of ice (W/m.K)
k_s	thermal conductivity of brine-spongy ice (W/m.K)
L_b	brine layer thickness (mm)
L_{Hb}	latent heat of fusion of brine (J/kg)
L_s	substrate thickness (mm)
M_s	mass of ice accumulated on the substrate of spongy ice (kg)
$M_{s,5‰}$	mass of ice accumulated on the substrate of spongy ice with the salinity of 5‰ (kg)
$M_{s,\,-5°C}$	mass of ice accumulated on the substrate of spongy ice with the temperature of −5°C (kg)
m	number of the node of solid phase closest to the phase interface (—)
n	number of the discretized grids (—)
$\dot{Q}_g'''$	rate of volumetric heat generation (J/m^3.s)
S	overall salinity (‰)
S_b	salinity of brine (‰)
T	temperature (°C)
T_i	initial temperature (°C)
T_{b1}	first interpolated temperature of liquid phase (°C)
T_{b2}	second interpolated temperature of liquid phase (°C)
T_f	freezing temperature (°C)
T_{s1}	first interpolated temperature of solid phase (°C)
T_{s2}	second interpolated temperature of solid phase (°C)
t	time (s)
t_0	initial time (s)
V_b	volume of brine in the element of brine-spongy ice (m^3)
V_{Fb}	volume fraction of brine (—)
V_i	volume of ice in the element of brine-spongy ice (m^3)
V_s	volume of the element of brine-spongy ice (m^3)
X	phase interface position (m)
α_{cb}	coefficient of the equation of specific heat capacity of brine (—)
α_{ci}	coefficient of the equation of specific heat capacity of ice (—)
α_{kb}	coefficient of the equation of thermal conductivity of brine (—)
α_{ki}	coefficient of the equation of thermal conductivity of ice (—)
α_{LHb}	coefficient of the equation of latent heat of fusion of brine (—)
α_{sb}	coefficient of the equation of salinity of brine (—)
$\alpha_{\rho b}$	coefficient of the equation of density of brine (—)
$\alpha_{\rho i}$	coefficient of the equation of density of ice (—)
$\Delta Q'''_g$	variation of volumetric heat generation (J/m^3)
Δt	time interval (s)
ΔV_{Fb}	variation of volume fraction of brine (—)
ΔV_i	variation of volume of ice (m^3)
Δx	grid length (m)
δ	freezing fraction of solidification (—)
ρ_b	density of brine (kg/m^3)
ρ_e	equivalent density of brine-spongy ice (kg/m^3)
ρ_i	density of ice (kg/m^3)
ρ_s	density of brine-spongy ice (kg/m^3)

rapid freezing phenomenon, is a common mechanism of ice accretion on vessels and offshore structures (Kulyakhtin and Tsarau, 2014). A cold substrate of spongy ice is capable of absorbing the released latent heat of fusion that can control the rate of ice accretion (Kulyakhtin et al., 2016; Saha et al., 2016; Wettlaufer, 1992a).

A substrate of spongy ice acts as a thermal capacitor in the process of ice accretion. In the cooling process, spongy ice becomes colder and the trapped brine pockets start to freeze and join the pure ice (Kulyakhtin et al., 2016; Kulyakhtin and Tsarau, 2014; Makkonen, 1987). The amount of freezing of the brine pockets depends on the corresponding cooling process and the final temperature of the spongy ice. In the freezing process, the spongy substrate absorbs the latent heat of fusion released by the phase interface (Blackmore et al., 2002; Kulyakhtin and Tsarau, 2014).

The freezing phenomenon of fresh water in various cases has been widely investigated. For fresh water, the solidified phase is solid ice and the phase interface is located between the solid ice and water (Brakel et al., 2007; Cao et al., 2016; Kong and Liu, 2015). The freezing of brine is more complicated than the freezing of fresh water. Salt rejection and brine trapping cause the creation of spongy ice. Trapped brine pockets affect the freezing process. Variations in the salinity of brine change the quality of the accumulated ice. Cooling rates affect the mechanism of brine trapping. The phase interface and the spongy substrate are two-phase media that are affected by the rate of heat exchange in their vicinity (Dupont et al., 2015; Heimbach et al., 2010; Losch et al., 2010; Wettlaufer, 1992a, 1992b; Worster and Wettlaufer, 1997).

Common finite-difference methods for solving a freezing problem involve discretized equations based on fixed-grid discretization (Caldwell and Kwan, 2004; Chun and Park, 2000; Furenes and Lie, 2006). A challenge of using fixed-grid methods in solving a solidification problem is the moving phase interface on the vicinal nodes. The discretized values for the nodes close to the phase interface are calculated based on the properties of the phase interface, which is not fixed in space, and the other neighbor nodes. This creates some instability in the process of the numerical solution. The Method of Lines (MOL) is a common method for arranging the discretized equations for numerical iterations (Schiesser, 2012).

In this research, newly created spongy ice during a rapid freezing process of salt water is named *brine-spongy ice*, which is a mixture of brine pockets and pure ice. The brine pockets are trapped momentarily and do not have enough time to join together and create big brine pockets or drain channels. In this paper, the governing equation of heat transfer through brine-spongy ice is derived and the heat balance for the phase interface is obtained. A discretization method is developed for discretizing the liquid phase and solid phase by considering the moving phase interface. Numerical solutions report the results of the solidification process for rapid freezing of a brine layer on a substrate of brine-spongy ice. Experimental tests and measurements are compared with the model and the numerical solutions.

2. Freezing model

Brine-spongy ice is a two-phase medium exists for a wide range of temperatures which is possible in Arctic and cold regions. For a salinity of salt water close to that of seawater, about 35‰, the temperature range is approximately between −2 °C and −21 °C. For temperatures