International Journal of Heat and Fluid Flow 000 (2017) 1–8

Contents lists available at ScienceDirect

International Journal of Heat and Fluid Flow

journal homepage: www.elsevier.com/locate/ijhff

High-order structure functions for passive scalar fed by a mean gradient

M. Gauding, L. Danaila*, E. Varea

CORIA UMR 6614, Université de Rouen, 76801 Saint Etienne du Rouvray, France

ARTICLE INFO

Article history:
Available online xxx

ABSTRACT

Transport equations for even-order structure functions are written for a passive scalar mixing fed by a mean scalar gradient, with a Schmidt number $Sc = 1$. Direct numerical simulations (DNS), in a range of Reynolds numbers $R_\lambda \in [88, 529]$ are used to assess the validity of these equations, for the particular cases of second-and fourth-order moments. The involved terms pertain to molecular diffusion, transport, production, and dissipative-fluxes. The latter term, present at all scales, is equal to: i) the mean scalar variance dissipation rate, $\langle\chi\rangle$, for the second-order moments transport equation; ii) non-linear correlations between χ and second-order moments of the scalar increment, for the fourth-order moments transport equation.

The equations are further analysed to show that the similarity scales (i.e., variables which allow for perfect collapse of the normalised terms in the equations) are, for second-order moments, fully consistent with Kolmogorov–Obukhov–Corrsin (KOC) theory. However, for higher-order moments, adequate similarity scales are built from $\langle\chi^n\rangle$. The similarity is tenable for the dissipative range, and the beginning of the scaling range.

1. Introduction

Fully developed turbulence is characterized by a large range of length scales, varying from the so-called integral length scale l_t, at which large velocity fluctuations occur on average, down to the smallest scale, the so-called Kolmogorov or dissipation scale η, at which turbulent fluctuations are dissipated. Until now, most understanding of turbulent flows has been gained from Kolmogorov's scaling theory (Kolmogorov, 1941a, 1941b), which was later extended by Obukhov (1949) and Corrsin (1951) to passive scalars advected by a turbulent velocity field. The Kolmogorov–Obukhov–Corrsin (hereafter, KOC) theory postulates that, under the condition of sufficiently high Reynolds numbers, the small scales of the flow decouple from the large scales. The understanding is that there is a steady cascade from the large scales to the smallest scales, where the energy transfer rate is equal to the mean energy dissipation rate $\langle\varepsilon\rangle$. Kolmogorov hypothesized that the small scales should depend only on two parameters, namely the viscosity ν and the mean energy dissipation $\langle\varepsilon\rangle$. Because only two quantities with different physical units are involved, this was viewed as a claim of universality. If the notion that the small-scale motion is universal was strictly valid, then there would be realistic hope for a statistical theory of turbulence.

The local and non-local phenomena which are inherent to turbulent flows can be analyzed by the moments of the scalar increment $\Delta\phi$, the so-called structure functions, defined by

$$S_p(\boldsymbol{x}, \boldsymbol{r}) = \langle(\phi(\boldsymbol{x}+\boldsymbol{r}) - \phi(\boldsymbol{x}))^p\rangle = \langle(\Delta\phi)^p\rangle, \quad (1)$$

where $\boldsymbol{r}$ is the separation vector between the two points and the angular brackets denote an ensemble-average. In statistically homogeneous turbulence, structure functions are independent of the position $\boldsymbol{x}$. Yaglom (1949) presented an exact transport equation for the second order scalar structure function in homogeneous isotropic turbulence, where the separation distance r (the modulus of vector $\boldsymbol{r}$) is the independent variable. i.e.

$$\frac{\partial}{\partial t}\langle(\Delta\phi)^2\rangle + \frac{\partial}{\partial r_i}\langle(\Delta u_i)(\Delta\phi)^2\rangle = 2D\frac{\partial^2}{\partial r_i^2}\langle(\Delta\phi)^2\rangle - 2\langle\chi\rangle, \quad (2)$$

where $\Delta u_i = u_i(\boldsymbol{x}+\boldsymbol{r}) - u_i(\boldsymbol{x})$ denotes the velocity increment and D the molecular diffusivity. Eq. (2) uses Einstein's summation convention, which implies summation over indices appearing twice. The mean scalar dissipation is defined as,

$$\langle\chi\rangle = 2D\left\langle\left(\frac{\partial\phi}{\partial x_i}\right)^2\right\rangle. \quad (3)$$

* Corresponding author.
E-mail address: danaila@coria.fr (L. Danaila).

http://dx.doi.org/10.1016/j.ijheatfluidflow.2017.05.009
0142-727X/

Provided that the Reynolds number is high enough, Eq. (2) has two separate analytical solutions. One in the dissipative range for $r \to 0$, where the diffusive term and the scalar dissipation balance, i.e.

$$\langle(\Delta\phi)^2\rangle = \frac{\langle\chi\rangle}{6D} r^2, \tag{4}$$

and one for the inertial range for $\eta \ll r \ll l_t$, where the transport term balances the scalar dissipation, i.e.

$$\langle(\Delta u_L)(\Delta\phi)^2\rangle = -\frac{2}{3}\langle\chi\rangle r, \tag{5}$$

with Δu_L being the longitudinal velocity increment in the direction of $\boldsymbol{r}$. These two results are of high significance. They are both exact and were derived from first-principles only under the assumptions of (local) isotropy and (local) homogeneity.

The paper is devoted to the analysis of S_p for $p = 2$ and $p = 4$, in the context of transport equations obtained from the first principles, by considering a restricted number of additional hypotheses, such as self-preservation, cf. Danaila and Mydlarski (2001). Section 2 describes the main characteristics of the performed direct numerical simulations (DNS), on which the analysis is based. In Section 3 we present the theory of generalized scalar scale-by-scale structure functions for any even order. Next, we develop in Section 4 similarity scales based on the scale-by-scale budget equations for the second and fourth order moments. The similarity scales are then justified by using data from DNS with different Taylor length-scale based Reynolds numbers between 88 and 529. Concluding remarks are given in Section 5.

2. Direct numerical simulations

We study a passive scalar advected by a statistically homogeneous isotropic and incompressible turbulent velocity field. In the present study, we consider a passive scalar with unity Schmidt number $Sc = \nu/D$, so that the kinematic viscosity ν equals the molecular diffusivity D. A uniform mean gradient Γ is imposed on the scalar field in x_2-direction. The mean gradient injects continuously energy into the scalar field and keeps statistics in a statistically steady state. The instantaneous scalar field can be decomposed in a mean part Γx_2 and a scalar fluctuation ϕ, namely

$$\Phi = \Gamma x_2 + \phi\,. \tag{6}$$

The scalar fluctuations ϕ are statistically homogeneous and obey the equation

$$\frac{\partial\phi}{\partial t} + u_i\frac{\partial\phi}{\partial x_i} = D\frac{\partial^2\phi}{\partial x_i^2} - \Gamma u_2, \tag{7}$$

where t is the time, x_i the spatial coordinates, and u_i denote the velocity field.

The three-dimensional incompressible Navier–Stokes equations in the vorticity formulation are solved together with Eq. (7) by a dealiased pseudo-spectral approach, cf. Canuto et al. (1988). Temporal integration is carried out by a second order semi-implicit Adams-Bashforth/Crank–Nicolson method. The integration domain is a triply periodic cube with length 2π. An external stochastic forcing, see Eswaran and Pope (1988), is applied to the velocity field to maintain a statistically steady state. The forcing is statistically isotropic and limited to low wave-numbers so that the small scales are not affected by the forcing scheme. The simulations have been carried out with an in-house hybrid MPI/OpenMP parallelized simulation code on the supercomputer JUQUEEN at research center Jülich, Germany.

Characteristic parameters of the DNS are presented in Table 1, where N denotes the number of grid points along one coordinate axis, Re_λ the Taylor based Reynolds number, $\langle k\rangle$ the mean kinetic energy, $\langle\varepsilon\rangle$ the mean energy dissipation, $\langle\phi^2\rangle$ the mean scalar variance, $\langle\chi\rangle$ the mean scalar dissipation, and $\langle u_2\phi\rangle$ the mean scalar flux. The production of scalar variance is $-2\langle u_2\phi\rangle\Gamma$. Ensemble-averages are denoted by angular brackets and are computed over the whole computational domain due to homogeneity and over a time frame t_{avg} due to stationarity. The number of analyzed ensembles is in the range between $M = 6$ for case R5 up to $M = 189$ for case R0. Resolving the smallest scales by the numerical grid is important for the accuracy of the DNS. To ensure an appropriate resolution of the smallest scales, the number of grid points has been increased to as high as $4096 \times 4096 \times 4096$ for case R5. Following Ishihara et al. (2007) and Donzis et al. (2005), a resolution condition of $\kappa_{max}\eta > 2.5$ is maintained for all cases to accurately compute high-order statistics. Here, κ_{max} denotes the highest resolved wavenumber and $\eta = \nu^{3/4}\langle\varepsilon\rangle^{-1/4}$ denotes the Kolmogorov length scale. Further details about the DNS are presented by Gauding et al. (2015) and Peters et al. (2016).

Table 1
Summary of different DNS cases. Reynolds number variation between $Re_\lambda = 88$ and $Re_\lambda = 529$.

	R0	R1	R2	R3	R4	R5
N	512^3	1024^3	1024^3	2048^3	2048^3	4096^3
Re_λ	88	119	184	215	331	529
ν	0.01	0.0055	0.0025	0.0019	0.0010	0.00048
$\langle k\rangle$	11.15	11.20	11.42	12.70	14.35	23.95
$\langle\varepsilon\rangle$	10.78	10.52	10.30	11.87	12.55	28.51
$\langle\phi^2\rangle$	1.95	1.89	1.94	2.47	2.25	2.41
$\langle\chi\rangle$	3.92	3.90	4.01	5.00	4.76	6.78
$-2\Gamma\langle u_2\phi\rangle$	3.93	3.98	4.03	4.95	4.79	5.76
t_{avg}/τ	100	30	30	10	10	2
M	189	62	61	10	10	6
$\kappa_{max}\eta$	3.93	4.99	2.93	4.41	2.53	2.95

3. Scale-by-scale transport equations for even order structure functions

Starting from Eq. (7), a transport equation for the even moments of the scalar increment can be derived by using a similar procedure as presented in Danaila et al. (1999) and Hill (2001). For homogeneous turbulence this equation reads

$$\frac{\partial}{\partial t}\left\langle(\Delta\phi)^{2n}\right\rangle(\boldsymbol{r}) + \underbrace{\frac{\partial}{\partial r_i}\left\langle(\Delta u_i)(\Delta\phi)^{2n}\right\rangle(\boldsymbol{r})}_{-Tr_{2n}} + \underbrace{2n\Gamma\left\langle(\Delta u_2)(\Delta\phi)^{2n-1}\right\rangle(\boldsymbol{r})}_{-Pr_{2n}} = J_{2n}(\boldsymbol{r}), \tag{8}$$

where J_{2n} are the molecular-diffusion terms, i.e.

$$J_{2n}(\boldsymbol{r}) = nD\left\langle(\Delta\phi)^{n-1}\left[\frac{\partial^2(\Delta\phi)}{\partial x_i'^2} + \frac{\partial^2(\Delta\phi)}{\partial x_i^2}\right]\right\rangle. \tag{9}$$

The term J_{2n} is a function of $\boldsymbol{r}$ and remains finite even when D tends to zero. Eq. (8) is exact, which means that it is derived from the governing equations without any approximation beside of incompressibility and homogeneity. It is convenient to decompose $J_{2n}(\boldsymbol{r})$ by partial integration, i.e.

$$J_{2n}(\boldsymbol{r}) = \underbrace{2D\frac{\partial^2}{\partial r_i^2}\langle(\Delta\phi)^{2n}\rangle}_{D_{2n}} - \underbrace{n(2n-1)\langle(\Delta\phi)^{2n-2}(\chi(\boldsymbol{x})+\chi(\boldsymbol{x}+\boldsymbol{r}))\rangle}_{E_{2n}}. \tag{10}$$

The terms of Eqs. (8) and (10) can be physically interpreted. The first term on the left-hand side is the temporal change of the moments of the scalar increment. The second term Tr_{2n} is a

Please cite this article as: M. Gauding et al., High-order structure functions for passive scalar fed by a mean gradient, International Journal of Heat and Fluid Flow (2017), http://dx.doi.org/10.1016/j.ijheatfluidflow.2017.05.009