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# Taylorian diffusion in mildly inhomogeneous turbulence

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ABSTRACT

Taylor's theory of dispersion was extended to produce estimates of the far-field growth rate of the plume of a passive scalar in grid turbulence (GT) and in uniformly sheared flow (USF), both of which evolve in the streamwise direction. Expressions for the evolution of the plume width relative to the integral length scale of turbulence were also derived. The predictions of plume growth rate were tested against available data in both of these types of flows and were found to be accurate in an extensive region of USF and compatible with an extrapolated trend in GT, in which the available data did not extend very far from the scalar source. Although in both cases the measured half-width of the plume was comparable in magnitude to the streamwise integral length scale of the turbulence, the far-field approximation seemed to hold.



## 1. Introduction

An admixture released from a concentrated source into a carrier fluid is mixed with its surroundings by molecular motions at a mixing rate that is proportional to the molecular diffusivity of the system. Nevertheless, molecular diffusion is greatly enhanced by turbulent motions, because turbulence increases drastically the interfacial area between regions of fluid with different concentrations.[1] Very close to the source, diffusion is only affected by molecular actions, irrespectively of whether the flow is laminar or turbulent, but, away from it, turbulent diffusion is overwhelmingly more effective than molecular diffusion. In his seminal work on turbulent diffusion, Taylor (1922) computed the rate of dispersion of fluid particles in one-dimensional, zero-mean, stationary and homogeneous turbulence. First he expressed the variance of the particle displacement $X$ at time $t$ following its release in terms of the Lagrangian velocity variance $\langle v^2\rangle$ (angle brackets denote ensemble averaging) and the Lagrangian velocity auto-correlation coefficient $R(\xi)$ as

$$\langle X^2(t)\rangle = 2\langle v^2\rangle \int_0^t (t-\xi)R(\xi)d\xi . \tag{1}$$

He further distinguished between dispersion at relatively short times, when $R(\xi)\approx 1$, and dispersion at relatively long times, when $R(\xi)\approx 0$. The interest of our work focuses on long-time diffusion and so the remainder of this article will deal only with this regime. Taylor's theory leads directly to an asymptotic expression for long-time particle dispersion as

$$\frac{1}{2}\frac{d\langle X^2(t)\rangle}{dt} \approx \langle v^2\rangle \mathscr{T} \ , \ t \gg \mathscr{T}, \tag{2}$$

where the Lagrangian integral time scale is defined as $\mathscr{T} = \int_0^\infty R(\xi)d\xi$ and the product $D=\langle v^2\rangle\mathscr{T}$ is known as the *turbulent diffusivity*. Taylor's analysis was one-dimensional, but its extension to three-dimensional homogeneous turbulence is straightforward (Batchelor, 1949). For convenience in estimating diffusion properties from measurements in approximately stationary and homogeneous turbulence, one may replace ensemble averages by time averages. One may also estimate the Lagrangian velocity variance and the Lagrangian time scale from the Eulerian velocity standard deviation $u'$ and the Eulerian integral length scale $L$ (Corrsin, 1975) as $\langle v^2\rangle \approx u'^2$ and $\mathscr{T}\approx L/u'$. Then, the turbulent diffusivity may be also be approximated as $D\approx u'L$.

Taylor's theory of dispersion has also been adapted to the description of turbulent diffusion of a passive scalar injected from a point or line source in stationary and homogeneous turbulence that is convected by a mean stream with a uniform velocity $\overline{U}_1$ (overbars denote time averaging) (Arya, 1999). For a point source, or a line source aligned with the $x_3$ axis, one may ascertain that the far-field plume would have a Gaussian-shaped mean profile (see Fig. 1 for a sketch) with a half-width

$$\sigma \approx \left(2u_2' L_{22,2}\Delta x_1/\overline{U}_1\right)^{1/2}, \ \sigma \gg L_{22,2}, \tag{3}$$

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[1] The following discussion also applies to changes of temperature of packets of fluid released into cooler or warmer bulk fluid, in which case the molecular diffusivity should be substituted by the thermal diffusivity.

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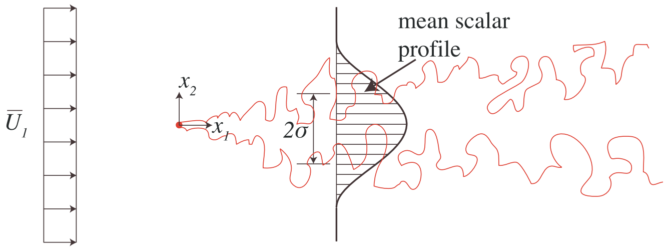


**Fig. 1.** Sketch of a passive scalar plume released from a discrete source in unidirectional, stationary and homogeneous turbulent flow.

where $\Delta x_1$ is the streamwise distance from the source, $u'_2$ is the standard deviation of the transverse velocity fluctuations and $L_{22,2}$ is the transverse integral length scale of the transverse velocity fluctuations.

Estimating the width of scalar plumes is of great practical importance in many environmental and industrial applications and the use of a simple equation like (3) would be an attractive approach, provided of course that it is sufficiently accurate. As mentioned previously, this relationship applies strictly to the far fields of stationary and homogeneous turbulence, and no turbulent flow, even among those generated in the laboratory, meets exactly such conditions. It seems, therefore, worthwhile to seek types of turbulence that meet approximately the conditions for the validity of Eq. (3) and so would be possible candidates for an extension of Taylor's theory. To distinguish such extensions from Taylor's original theory of dispersion, we will hereafter denote them by the term *Taylorian diffusion*. To qualify for this name, a Taylorian diffusion analysis must be based on two main conditions: first, the inhomogeneity of the turbulence must be sufficiently mild ("homogeneity condition") and, second, the plume width must be large compared to the local integral length scale ("far-field condition"). The far-field condition can be readily verified from experimental data. One way to ensure that the homogeneity condition is satisfied would be to confirm that changes in turbulent kinetic energy and integral length scale over a streamwise or transverse distance that is equal to the local length scale are small by comparison to the values of the corresponding properties, spatially averaged over the same distance. In such cases, one may argue that Eq. (3) would still lead to an estimate of plume width in terms of the streamwise distance $\Delta x_1$, provided that the local values of $u'_2$ and $L_{22,2}$ are permitted to evolve, rather than being fixed, as in Taylor's theory. Thus, the problem of estimating a Taylorian plume width would be reduced to finding suitable expressions for the evolutions of these properties.

The most promising Taylorian diffusion candidates seem to be canonical flows that evolve in a self-similar manner far away from their origin. In such flows, one may use the turbulent kinetic energy $k$ as a surrogate for $u'^2_2$ and the streamwise integral length scale $L_{11,1}$ as a surrogate for $L_{22,2}$, because $L_{11,1}$ is the only one reported in most cases by experimentalists. $k$ can be expressed as a function of streamwise distance, either empirically or by solving the simplified turbulent kinetic energy equation. The latter equation may be also used to derive a corresponding expression for the turbulent kinetic energy dissipation rate $\varepsilon$. Unfortunately, there is no independent analytical way to determine the streamwise evolution of the integral length scale; a common approach is to estimate it by assuming constancy of the *dissipation parameter* (Taylor, 1935)

$$C_\varepsilon = \frac{\varepsilon L_{11,1}}{(2k/3)^{3/2}} \approx \text{const.} \tag{4}$$

It has been amply demonstrated (Vassilicos, 2015; Nedić and Tavoularis, 2016b; 2016a; Nedić et al., 2017) that (4) holds in far downstream regions of several canonical flows and so its range of validity is likely to overlap with the Taylorian far-field condition. There is also growing evidence that several canonical flows have extensive upstream regions in which (4) is not valid (Nedić et al., 2013; Dairay et al., 2015; Vassilicos, 2015; Goto and Vassilicos, 2015; Obligado et al., 2016; Nedić and Tavoularis, 2016b; Goto and Vassilicos, 2016; Nedić and Tavoularis, 2016a; Nedić et al., 2017), and, in some cases, also intermediate regions where the dissipation parameter may be described as a power function of the local turbulence Reynolds number $Re_\lambda = \lambda(2k/3)^{1/2}/\nu$ ($\lambda$ is the Taylor microscale and $\nu$ is the kinematic viscosity of the fluid), namely as

$$C_\varepsilon \propto Re_\lambda^\alpha. \tag{5}$$

It is obvious that (4) is a special case of (5), corresponding to $\alpha = 0$. For a particular turbulent flow to have a Taylorian diffusion region, it must satisfy the two Taylorian conditions. Turbulent boundary layers and free shear flows (wakes, jets and mixing layers) have integral length scales that are comparable to their width and so a plume width would never satisfy the far-field condition in such flows. This leaves flows which are (ideally) unbounded as only possible Taylorian candidates. The only two such flows that are known to exist are grid-generated, nearly isotropic turbulence (GT) and uniformly sheared, highly anisotropic turbulence (USF). Away from their origins, both GT and USF are approximately homogeneous on transverse planes but inhomogeneous in the streamwise direction, although mildly so. They are also known to have extensive self-similar regions. In Section 2 we will derive theoretical predictions of Taylorian plume widths in these two flows and in Section 3 we will test these predictions against available experimental results.

## 2. Predictions of Taylorian diffusion analysis

### 2.1. Decaying grid turbulence

Sufficiently far downstream of a grid, the turbulence would essentially be transversely homogeneous and the turbulent kinetic energy would decay as

$$k \propto (x_1 - x_{10})^{-m}, \tag{6}$$

where the origin of the coordinate system is fixed on the grid and $x_{10}$ is the location of an empirical effective origin. Far downstream of the grid, turbulence production by mean shear would be negligible and the turbulent kinetic energy equation may be simplified to $0.5\overline{U}_1(dk/dx_1) \approx -\varepsilon$ from which one may also derive a dissipation decay law as

$$\varepsilon \propto (x_1 - x_{10})^{-(m+1)}. \tag{7}$$

Considering that the Taylor microscale $\lambda$ is defined in terms of $k$ and $\varepsilon$ as $\varepsilon = 10\nu k/\lambda^2$, it is easy to show that

$$\lambda \propto (x_1 - x_{10})^{1/2}, \tag{8}$$

from which one may also derive power laws for the turbulent Reynolds number as

$$Re_\lambda \propto \lambda k^{1/2} \propto (x_1 - x_{10})^{(1-m)/2} \tag{9}$$

and for the dissipation parameter as

$$C_\varepsilon \propto (x_1 - x_{10})^{\alpha(1-m)/2}. \tag{10}$$

Finally, combining (4), (6) and (7), one may derive a power law for the integral length scale as

$$L_{11,1} \propto C_\varepsilon (x_1 - x_{10})^{(2-m)/2} \propto (x_1 - x_{10})^{(2-m)/2+\alpha(1-m)/2}. \tag{11}$$

When attempting to predict plume growth in decaying turbulence, one needs to overcome an obvious contradiction. The decay law is referenced to a distance from an effective origin at $x_{10}$,