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Effect of the hydrodynamic pressure on shaft torque for a 4-blades vane rheometer

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ABSTRACT

Vane type geometries are often used in rheometers to avoid slippage between the sample and the solid boundaries. Thus in commercial firms, universities and research institutes, vane type geometries are often applied to obtain accurate isothermal flow parameters. Recently, a detailed analysis of the complex flow phenomena inside such device was made using the finite element method (Nazari et al., 2013). In particular, the shaft torque was calculated at different angular velocities and compared to experimental data. Here, this work is repeated by using the finite volume method. In addition to this, the effect of hydrodynamic pressure on the shaft torque is analyzed separately and compared to the effect generated by viscous stress. This analysis is done for the two Newtonian fluids reported in Nazari et al. (2013), as well as for two cases of viscoplastic Bingham fluids. Including this, mesh independence is also analyzed. The outcome of the overall analysis is that the effect of hydrodynamic pressure constitutes about 2/3 of the shaft torque, leaving only about 1/3 of the torque to viscous stress.

1. Introduction

As reported in Nazari et al. (2013), the vane rheometer consists of an impeller (the rotor) rotating in a baffles-cylinder geometry (the stator). The objective of this geometrical configuration is to eliminate slip between the sample and the solid boundaries of the rheometer (Nazari et al., 2013). However, with rotation, the impeller's vane blades will both push and drag the fluid, resulting in non-uniform hydrodynamic pressure exerted on the blades. Thus, in addition to the viscous shear stress, this pressure will influence the measured torque registered by the rheometer (i.e. influence the shaft torque). As such, the main question becomes: *How significant is this influence?* Is there a direct relationship between the output of the vane rheometer and the fluid shear viscosity η, or will the effect of hydrodynamic pressure distort or damage this relationship? Or more to the point, can the results of the vane rheometer be trusted to give an accurate information about the fluid shear viscosity? The main objective of the current study is to answer this, not only relative to Newtonian fluids, but also relative to non-Newtonian Bingham fluids.

The benefits and faults of the vane rheometer for different applications and conditions, e.g. concentrated suspensions, polymer systems, secondary flow regimes and so forth, is well reported in Nazari et al. (2013) and thus such will not be repeated here. In addition to this, the Reynolds number *Re* and justification for laminar flow analysis is well treated in Nazari et al. (2013) and thus is neither addressed again here.

The CFD software used in Nazari et al. (2013) was the COMSOL Multiphysics 4.1. However in this work, the OpenFOAM 2.1.1 is utilized. It is licensed under the GNU General Public License (GNU GPL) and is available at www.openfoam.org, without charge or annual fee of any kind. The benefits of having a GNU GPL licensed code over a closed commercial code, is that the user has always a full access to the source code, without any restriction, either to understand, correct, modify or enhance the software. OpenFOAM is written in C++. As such, an object-oriented programming approach is used in the creation of data types (fields) that closely mimics those of mathematical field theory (Weller et al., 1998). For the code parallelization and communication between processors, the domain decomposition method is used with the Message Passing Interface, or MPI (Berberović et al., 2010).

For the specific OpenFOAM solver used in this work, namely the MRFSimpleFoam, the three dimensional momentum equation as well as the continuity equation (see Section 3.3) are solved in parallel to obtain the velocity and hydrodynamic pressure profiles throughout the geometry. More precisely, the pressure velocity coupling is handled with a Semi-Implicit Method for Pressure-Linked Equations (SIMPLE) procedure (Versteeg and Malalasekera, 2007), using a modified Rhie–Chow interpolation for cell centered

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Nomenclature

$\mathbf{i}_z$	unit vector in the axis of the shaft (i.e. here the z-axis) (–) (Section 3.4)
p	hydrodynamic pressure (Pa) (Sections 3.3 and 3.4)
$\mathbf{T}$	extra stress tensor (Pa) (Eq. (6))
T	total shaft torque, or "total torque" ($T = T_\eta + T_p$) (N m) (Section 3.4)
$\mathbf{T}_p$	torque by hydrodynamic pressure (N m) (Eq. (10))
T_p	$= \mathbf{T}_p \cdot \mathbf{i}_z$ shaft torque by hydrodynamic pressure, or "pressure–torque" (N m) (Section 3.4)
$\mathbf{T}_\eta$	torque by viscous stress (N m) (Eq. (9))
T_η	$= \mathbf{T}_\eta \cdot \mathbf{i}_z$ shaft torque by viscous stress, or "viscous–torque" (N m) (Section 3.4)
$\mathbf{U}$	velocity (in the inertial reference frame) (m/s) (Section 3.3)

Greek letters

$\dot{\gamma}$	shear rate (s^{-1}) (Section 3.1)
$\dot{\boldsymbol{\varepsilon}}$	rate-of-deformation tensor (s^{-1}) (Section 3.1)
η	shear viscosity (also, apparent viscosity) (Pa s) (Section 3.2)
μ	plastic viscosity (Pa s) (Section 3.2)
ρ	density (kg/m^3) (Eqs. (7) and (8))
$\boldsymbol{\sigma}$	(total) stress tensor (also, constitutive equation) (Pa) (Section 3.1)
τ_0	yield stress (also, yield value; cf. British Standard BS 5168:1975) (Pa) (Section 3.2)
ω	shaft angular velocity (and of the 4 blades connected to it) (rpm) (Sections 3.3 and 4)

Symbols

$\sim$	in the order of magnitude of (as in $55 \sim 10^2$)
$\approx$	approximate equal to (as in $55 \approx 56$)

data storage (Karrholm, 2006). Relative to the MRFSimpleFoam, the SIMPLE consists of the following steps: (1) Set up the discretized momentum predictor with relaxation. (2) Solve the discretized momentum predictor. (3) Compute the cell face fluxes. (4) Solve the pressure equation. (5) Correct and adjust the flux at cell faces in a such manner that it is guaranteed conserved. (6) Apply under relaxation to the pressure result. (7) Correct the velocities on the basis of the new (relaxed) pressure field. (8) Update the boundary conditions. (9) Repeat until the convergence criteria are satisfied.

2. Geometry, mesh and boundary conditions

The specific vane rheometer under consideration is as described in Nazari et al. (2013) and is a controlled stress rheometer. As shown in the left illustration of Fig. 1, its rotating geometry consists of a shaft (i.e. a rod) connected to four blades. The center-left and center-right illustrations show how the stationary part consists of four baffles connected to a cylinder (i.e. to a cub). The right illustration demonstrates how the shaft–blades system is submerged into the baffles-cylinder system (the center-left illustration also shows this).

The thickness of each blade is 2.05 mm. The edge-to-edge diameter of the blades (including the shaft diameter) is 34.0 mm. The diameter of the shaft is 5.95 mm. The diameter of the cylinder is 49.8 mm, while it is $49.8 - 2 \cdot 3.7 = 42.4$ mm relative to the extremities of the baffles. The thickness of each baffle is 1.95 mm. The height of the (rotating) blades is 49.9 mm, while the height of the cylinder is 60.1 mm. All these geometric values are obtained from Fig. 1 in Nazari et al. (2013).

To investigate the mesh dependency of the numerical result, four different mesh densities (or mesh resolutions) are used, namely 110,496, 316,056, 514,320 and 1,060,080 cells. The mesh is generated with a native OpenFOAM mesh utility called blockMesh, and the mesh quality is checked with another OpenFOAM utility, named checkMesh. For all the mesh cases, more than 96% of the cells are hexahedra. The remaining cells consist of prisms, tetrahedra and polyhedra.

The no-slip boundary condition (i.e. the Dirichlet boundary condition) is used at all wall boundaries. However, at the top of the rheometer (i.e. at/near the boundary between atmosphere and liquid), the boundary condition consists of $\mathbf{i}_z \cdot \nabla \mathbf{U} = \partial \mathbf{U}/\partial z = 0$ (i.e. the Neumann boundary condition).

3. Theoretical background

3.1. Bingham model

In this work, the two standard Newtonian fluids described in Nazari et al. (2013) are used in the CFD simulations. But including these two cases, two Bingham viscoplastic fluids are also applied here. With this, the constitutive equation used consists of the Generalized Newtonian Model (Tanner and Walters, 1998), or in short GNM. The GNM is given by $\mathbf{T} = 2\eta\dot{\boldsymbol{\varepsilon}}$, where the terms $\mathbf{T}$ and η are the extra stress tensor and the shear viscosity, respectively (Barnes et al., 1989). The term $\dot{\boldsymbol{\varepsilon}} = \frac{1}{2}(\nabla \mathbf{U} + (\nabla \mathbf{U})^{\mathrm{T}})$ is known as the rate-of-deformation tensor and $\mathbf{U}$ represents the inertial velocity (Malvern, 1969; Mase, 1970).

Since the Bingham model converges to the Newtonian model as the yield stress τ_0 approaches zero, only the former is treated here: In Oldroyd (1947a,b), a von Mises yield criterion is used to describe the Bingham fluid. Using such approach (with the above mentioned GNM), the flow behavior of the Bingham model can be described with

$$\mathbf{T} = 2\left(\mu + \frac{\tau_0}{\sqrt{2\dot{\boldsymbol{\varepsilon}} : \dot{\boldsymbol{\varepsilon}}}}\right)\dot{\boldsymbol{\varepsilon}} \quad \text{for} \quad \mathbf{T} : \mathbf{T}/2 \geqslant \tau_0^2, \tag{1}$$

$$\dot{\boldsymbol{\varepsilon}} = 0 \quad \text{for} \quad \mathbf{T} : \mathbf{T}/2 < \tau_0^2. \tag{2}$$

According to Eqs. (1) and (2), the yield surface is located by the condition $\mathbf{T} : \mathbf{T}/2 = \tau_0^2$. In the region where $\mathbf{T} : \mathbf{T}/2 < \tau_0^2$, the material behaves as a rigid solid, while in the region with $\mathbf{T} : \mathbf{T}/2 > \tau_0^2$, the material flows with a shear viscosity of $\eta = \mu + \tau_0/\sqrt{2\dot{\boldsymbol{\varepsilon}} : \dot{\boldsymbol{\varepsilon}}}$. In the above, the more correct approach would be using $\mathbf{S} : \mathbf{S}/2$ instead of $\mathbf{T} : \mathbf{T}/2$, where $\mathbf{S} = \sigma - (\mathrm{tr}(\boldsymbol{\sigma})/3)\mathbf{I}$. But in the next paragraph it will be shown that $\mathbf{S} = \mathbf{T}$ and thus either can be used. The term $\mathbf{S}$ is known as the deviator stress tensor and $\boldsymbol{\sigma}$ is the (total) stress tensor (Malvern, 1969; Mase, 1970). The latter is given by $\boldsymbol{\sigma} = -p\mathbf{I} + \mathbf{T}$, where $\mathbf{I}$ is the unit dyadic (Barnes et al., 1989).

Only incompressible material is treated in this research, meaning $\mathrm{tr}(\dot{\boldsymbol{\varepsilon}}) = \nabla \cdot \mathbf{U} = 0$. The material is also assumed isothermal, but the following derivation is not contingent on that. For incompressible fluid, the extra stress tensor $\mathbf{T}$ is also the deviator stress tensor $\mathbf{S}$. That is, $\mathbf{S} = \boldsymbol{\sigma} - (\mathrm{tr}(\boldsymbol{\sigma})/3)\mathbf{I} = -p\mathbf{I} + \mathbf{T} + p\mathbf{I} - (2/3)\eta\mathrm{tr}(\dot{\boldsymbol{\varepsilon}})\mathbf{I} = \mathbf{T}$. Therefore, the term $-\mathbf{T} : \mathbf{T}/2$ represents the second invariant of the deviator stress tensor $\mathrm{II}_{\mathbf{S}} = (\mathrm{tr}(\mathbf{S})\mathrm{tr}(\mathbf{S}) - \mathbf{S} : \mathbf{S})/2 = -\mathbf{S} : \mathbf{S}/2$. Hence, the von Mises shear stress is