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# Numerical study of the electro–thermo-convection in an annular dielectric layer subjected to a partial unipolar injection

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ABSTRACT

This article deals with the problem of electro–thermo-hydro-dynamic (ETHD) in a dielectric liquid placed between two horizontal isothermal coaxial cylinders and subjected to the simultaneous action of a thermal gradient and an electric field. The full set of equations describing the combined ETHD flow is directly solved using the control-volume method. The effect of electroconvective motion was investigated in details in the case of strong unipolar injection from the half bottom of the inner cylinder. We focused mainly on enhancing the heat transfer by unipolar injection of electric charge in the dead zone (bottom area) of the annular space.

The results show the emergence of a multicellular flow in this dead zone which enhances the heat transfer up to 170%. The flow structure in terms of stream lines, distribution of electric charge density and thermal field is highlighted. The effect of various system parameters in particular the radius ratio, the electric Rayleigh number is investigated too.

## 1. Introduction

Electro–thermo-convection in an insulating liquid layer subjected to unipolar injection is a very interesting problem because of its theoretical and practical importance. The Coulomb force applied by an electric field on any charge present in a dielectric liquid may cause fluid motion and a significant increase of heat transfer. At high enough applied electric fields in an insulating liquid, electric charge carriers are created at metallic/liquid interfaces (Atten, 1999), a process referred to as ion injection results from electrochemical reactions.

These last years, several theoretical and experimental studies were able to characterize the structure of the flow in an annulus (Atten and Elouadie, 1995; Agräit and Castellanos, 1990; Pei-chun, 2007; Deyirmenjian et al., 1997).

Agräit and Castellanos (1990) used the linear stability theory to discriminate the electric convective structures in the case of unipolar injection between coaxial cylinders of arbitrary radii. They considered the case of weak, strong and arbitrary finite injection either from inner or outer cylinder.

Later Deyirmenjian et al. (1997) were able to demonstrate experimentally the electroconvection phenomena. They showed the multicellular regime composed by counter-rotating vortices. They also exhibited a variety of interesting nonlinear effects.

Apart from these theoretical or experimental investigation means, numerical simulation arises as a very efficient alternative as it can provide a deeper insight of the flow structure and thus a better understanding of the physical mechanisms.

One crucial point in electro–thermo-convection simulations is the computation of the electric charge density, from the charge conservation equation, which is a major source of numerical difficulties. For this reason the computational resolution of the whole set of governing equations that model the electro–thermo-hydro-convection are rather rare in rectangular cavities (Traoré and Perez, 2012; Traoré et al., 2010; Koulova et al., 2013; Hassen et al., 2012) and quasi inexistent in the annular spaces (Lara et al., 1998; Tsai et al., 2007; Fernandes et al., 2012; Hassen et al., 2013).

Lara et al. (1998) studied the linear stability of an isothermal dielectric fluid subjected to unipolar injection inside an annulus. They worked on Taylor–Couette flow induced by the rotation of the outer or inner cylinder. The authors discussed the physical mechanisms associated with instabilities of steady flows. In Tsai et al. (2007) the authors have analyzed the stability of an isothermal liquid crystal flow when the emitter electrode was the inner cylinder. They defined the electric Nusselt number as an indicator which helps to determine whether the flow belongs to the conductive regime (where viscous dissipation effects dominate) or to the convective regime (where electric forces take the control).

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## Nomenclature

| | |
|---|---|
| $a$ | thermal diffusivity |
| $C$ | dimensionless number of the injection strength |
| $\vec{E}$ | electric field |
| $\vec{g}$ | acceleration of gravity |
| $K$ | ionic mobility |
| $M$ | dimensionless number which characterizes EHD properties of the liquid |
| $Pr$ | Prandtl number |
| $q$ | electric charge density |
| $r$ | radial coordinate |
| $r_c = \frac{(r_i+r_o)}{2}$ | radial coordinate of the center of the annular gap |
| $R$ | electric Reynolds number |
| $Ra$ | thermal Rayleigh number |
| $T$ | dimensionless electric Rayleigh number |
| $t$ | time |
| $\vec{U}$ | velocity |
| $V$ | electric potential |

*Greek symbols*

| | |
|---|---|
| $\Gamma = r_i/r_o$ | radius ratio |
| $\varepsilon$ | permittivity of the fluid |
| $\theta$ | dimensionless temperature |
| $\rho$ | density |
| $\varphi$ | angular coordinates |
| $\nu$ | kinematic viscosity |
| $\mu$ | dynamic viscosity |
| $\psi$ | stream function |
| $\omega$ | vorticity |

*Subscript*

| | |
|---|---|
| $i$ | inner cylinder |
| $o$ | outer cylinder |
| $r$ | radial |
| $\varphi$ | angular |

Fernandes et al. (2012) have performed some numerical simulations where the emitter electrode could be alternatively the inner or outer cylinder. They computed the critical electric Rayleigh number for the onset of electro-convection at various injection strength and cylindrical aspect ratio. They also showed the existence of three regimes: stationary, oscillating and chaotic according to the value of the electric Rayleigh number.

Quite recently Hassen et al. (2013) have conducted numerical simulations of 2D fluid-flow of dielectric liquid between two perfectly conducting coaxial cylinders under autonomous unipolar charge injection. The authors have investigated the effect of injection strength, electric Rayleigh number and annular aspect ratio on the onset of electroconvection for inner injection.

In this paper the study is focused on electro–thermo-convection phenomenon of a dielectric layer between two coaxial cylinders. The purpose of this study is to analyze the effect of both electric and thermal fields on the annulus dielectric liquid layer. To achieve this objective, we solve the whole set of partial differential equations associated with the development of the electro and thermo-convective instability in this particular configuration. This set of coupled equations includes the Navier–Stokes equations, the energy equation, the charge density conservation equation and Poisson equation for potential. The reminder of this paper is organized as follows. In the following section (Section 2) the governing equations as well as the numerical method are provided. Then in Section 3 numerical results are presented and the flow structure is finely detailed for the case of strong injection. The charge density and the temperature field distributions are analyzed too. In this section also we quantify the real impact of electric-convection on heat transfer by exhibiting the temporal and spatial variations of the thermal Nusselt number. Finally a conclusion is given in Section 4.

## 2. Mathematical formulation

### 2.1. Governing equations

As shown in Fig. 1, the geometry under consideration is an annulus bounded by two infinite concentric perfectly conducting cylinders filled with incompressible and perfectly insulating liquid of density $\rho$, permittivity $\varepsilon$, and kinematic viscosity $\nu$.

These horizontal metallic electrodes (Fig. 1) are assumed to be rigid and heat conducting cylinders that are maintained respectively at fixed temperatures $\theta_i$ and $\theta_o$ ($\theta_i > \theta_o$). The emitter electrode corresponding to the bottom half of the internal cylinder is held at potential $V_i$ and is the source of ions which are injected into the bulk. These ions are collected by the outer cylinder which is held at potential $V_o$. The injection of unipolar charges of mobility $K$ at the emitter is assumed ‘homogeneous’ and ‘autonomous’. This means that $q = q_i$ at all times, and hence the injection rate is not influenced by the electric field.

Furthermore, it is assumed that the flow in the system is laminar with no-slip boundary conditions ($U_r = U_\varphi = 0$) at the inner and outer radii of the annulus. As the current density is very low in dielectric liquids the Joule heating is safely neglected. Finally the Boussinesq assumption is adopted which considers all physico-chemical variables constant except the density in the buoyancy force.

The governing equations of such a problem are ruled by the electro–thermo-hydro-dynamic equations, which is a set of coupled partial differential equations including the Navier–Stokes equations, the energy equation, the charge density transport equation and Gauss's law for the electrostatic potential.

$$\vec{\nabla} \cdot \vec{U} = 0 \tag{1}$$

$$\rho \frac{d\vec{U}}{dt} = -\vec{\nabla} p + \mu \vec{\nabla}^2 \vec{U} + \rho \vec{g} \beta (\theta - \theta_0) + q\vec{E} \tag{2}$$

$$\rho C_p \frac{d\theta}{dt} = \lambda \vec{\nabla}^2 \theta \tag{3}$$

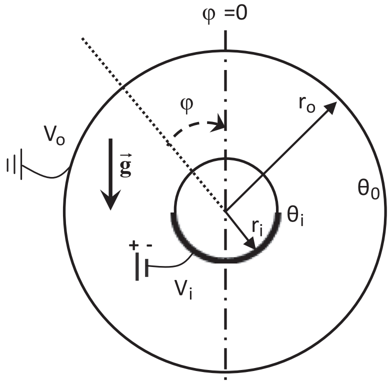

**Fig. 1.** The physical model.