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Numerical study on instabilities behind a circular disk in a uniform flow

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ABSTRACT

Instabilities behind a circular disk, which is normal to a uniform free-stream with aspect ratio of $\chi = 5$, are numerically studied in the range of Re from 115 to 1000. The onsets of three characteristic instabilities including natural vortex shedding, shear-layer instability and a very low frequency instability, which are associated with the Reynolds number, are reported. When Re < 152, the wake stays steady and no vortex shedding occurs. A regular bifurcation leading to two-thread wake is observed for Re from 120 to 152. When Re increases to 152, the hairpin vortices begin to be periodically shed in a regular pattern. A reflectional-symmetry-breaking (RSB) mode is observed for Re from 152 to 155, a standing-wave (SW) mode is captured for Re from 156 to 247, and an unsteady state with planar symmetry and non-zero mean lift is captured for Re from 248 to 264. It is found that the hairpin vortices are always shedding in a fixed orientation for RSB mode, while shedding from diametrically opposite orientations at SW mode and the unsteady state with planar symmetry and non-zero mean lift. When Re increases to 265, a low frequency instability attributed to the irregular rotation of vortex shedding location is observed. The hairpin vortices are periodically shed in an irregular pattern and a new '4L' vortex formation mode is identified. When Re increases to 650, the shear-layer instability starts to be detected in the shear layer. The critical Reynolds number is firstly identified in the range of 601–650. The cylindrical shear layer surrounding the vortex formation zone rolls up to form oblate ring vortices. The hairpin vortices break into pieces and small-scale vortices associated with the shear-layer instability are observed.

1. Introduction

Studies to date indicate that the wake behind bluff bodies such as a sphere or a circular disk is dominated by three instability mechanisms: natural vortex shedding, shear-layer instability, and a very low frequency instability. These instabilities in the wake behind bluff bodies are closely associated with the Reynolds number, $\mathrm{Re} = \frac{U_\infty d}{\nu}$, which is defined in terms of the main flow velocity U_∞ and the bluff body diameter d.

The instabilities in the sphere wake have been extensively studied. The onset of each instability in the sphere wake is summarized as following: (1) periodic hairpin vortex shedding is observed when Reynolds number increases to $\mathrm{Re}_1 = 277–300$; (2) a very low frequency instability occurs when Re increases to $\mathrm{Re}_2 = 350–375$; (3) when Re reaches $\mathrm{Re}_3 = 800$, the Kelvin-Helmholtz instability in the shear layer starts to develop. As reviewed by Kiya et al. (2001), the transitions of wake structure in the sphere wake, which are closely associated with the instabilities, can be described as follows. In the range of $\mathrm{Re}_1 < \mathrm{Re} < \mathrm{Re}_2$, hairpin vortices with a fixed orientation are periodically shed from the sphere and the wake structure is planar-symmetric. In the range of $\mathrm{Re}_2 < \mathrm{Re} < \mathrm{Re}_3$, the orientation of hairpin vortices tends to change from cycle to cycle, thus the wake becomes fully three-dimensional without planar-symmetry, and the cycle-to-cycle change becomes more and more irregular as Re increases. In the range of $\mathrm{Re} > \mathrm{Re}_3$, small-scale vortices, associated with the shear-layer instability, occur and cause a rapid distortion of the large-scale structure and eventually the wake becomes turbulent.

For the wake of a circular disk, however, the instabilities have been much less studied, though it is of great importance from both theoretical and practical points of view. Three instability mechanisms were experimentally confirmed by Berger et al. (1990), who investigated the near wake of a circular disk in the Reynolds number range $1.5 \times 10^4 \leqslant \mathrm{Re} \leqslant 3 \times 10^5$. However, most studies (before 2001) have been performed at Reynolds numbers of the order of 10^3–10^5, as reviewed by Kiya et al. (2001); the onset of each instability has not been investigated.

Recently, the wake of a circular disk in a uniform flow at low Reynolds numbers, i.e. $\mathrm{Re} \leqslant 350$, has been extensively investigated with the focus on the bifurcation scenarios. Studies indicate that the transition Reynolds number for each bifurcation is dependent

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on the aspect ratio, which is defined as $\chi = d/w$, where d and w are the diameter and the thickness of the disk, respectively. The first bifurcation was a regular transition leading to a steady planar-symmetric state, for all the investigated configurations including the flat disk ($\chi = \infty$) considered by Natarajan and Acrivos (1993), Fabre et al. (2008), Meliga et al. (2009), and disks of aspect ratio larger than one considered by Fernades et al. (2007) (χ = 2–10), Shenoy and Kleinstreuer (2008) ($\chi = 10$) and Auguste et al. (2010) ($\chi = 3$). The first bifurcation Reynolds number was found to be between 115 and 117 for the flat disk. The second bifurcation was found to lead to a three-dimensional state with hairpin vortex periodically shedding for a flat disk and a thin disk of aspect ratio $\chi = 10$. The second bifurcation Reynolds number was in the range 121–125.6 for the flat disk, and 155 for a thin disk ($\chi = 10$). Since the second bifurcation broke the planar-symmetry to form a three-dimensional wake, associated with a lift force oscillating in direction, this state was called reflectional-symmetry breaking (RSB) mode by Fabre et al. (2008). A third bifurcation leading to a periodic state with the planar-symmetry recovering was observed for a flat disk and the thin disk of aspect ratio $\chi = 10$. The lift force lies in a symmetry plane and oscillates about a zero mean in this state. The threshold Reynolds number was found at Re ≈ 140 (Fabre et al., 2008) and 143 (Meliga et al., 2009) for the flat disk, and at Re ≈ 172 for the thin disk ($\chi = 10$) (Shenoy and Kleinstreuer, 2008). This state was called standing-wave (SW) mode (Fabre et al., 2008; Meliga et al., 2009) or 'regular shedding of hairpin vortex with planar symmetry' (Shenoy and Kleinstreuer, 2008). The forth bifurcation was found to lead to a periodic three-dimensional flow with irregular rotation of the separation region for a thin disk ($\chi = 10$). A low frequency close to one third of the 'leading frequency' of the previous regimes was detected in this state. The critical Reynolds number was found at Re ≈ 280. An extensive parametric study dealing with oblate spheroids and flat cylinders in the range $\chi \in [1,\infty]$, Re ∈ [100,300] was recently carried out by Chrust et al. (2010). It basically confirmed the bifurcation scenarios summarized above and delimited the domain of existence of each state in the (χ, Re) plane. More detailed results for wake bifurcations of fixed bodies at the low Reynolds numbers were given with a review of Ern et al. (2012). From the studies above, it can be summarized that the instability due to natural vortex shedding starts to develop at the second bifurcation, and the low frequency instability occurs at the forth bifurcation. Unfortunately, the onset of the shear-layer instability has not been studied since these investigations have been only performed in the range of Re ⩽ 350. To our knowledge, there is still no investigation performed in the range of 350 < Re < 1000 for a circular disk wake in a uniform flow to date. One motivation of the present work is to explore this range so as to study the onset of the shear-layer instability.

Shear-layer instability studies for other flow configurations may provide some insight for a circular disk wake in a uniform flow. The characteristics of the shear-layer instability in the wake flow of a free annular jet across a circular disk were experimentally studied by Huang and Lin (2000) at Re < 2000. Five characteristic flow structures – laminar, subcritical, transitional, supercritical and turbulent modes, were identified in the shear layer evolving from the edge of the disk. Laminar mode occurred for Re < 390, at which no characteristic frequency was found. Subcritical mode occurred for 390 < Re < 640, at which the instability waves developed in the upstream area of the shear layer, evolved to form vortices as they traveled along the shear layer, and eventually broke up in the downstream area around the stagnation point. Transitional mode occurred for 640 < Re < 900, at which the unsteady motion of vortex shedding was observed in the shear layer. Supercritical mode occurred for 900 < Re < 1500, at which turbulent vortices shedding in the shear layer were observed. Turbulent mode occurred for Re > 1500, at which the periodicity in the shear layer was no longer found. Although this configuration is a little different from our case due to the effect of the blockage ratio, there may exist some similarities of the characteristics of the shear-layer instability in this Reynolds number range.

Considering the state of the art, the aim of this work is to numerically study the instabilities behind a circular disk, with an aspect ratio $\chi = 5$ in the range of Re from 115 to 1000. The Reynolds number defined in this paper is $Re = U_0 d/\upsilon$, where U_0 is the free stream velocity, d is the disk diameter, and υ is the kinematic viscosity. The onsets of the three characteristic instabilities (i.e., vortex shedding instability, low frequency instability, and shear-layer instability), as well as the transitions of the wake structures associated with the instabilities are investigated in detail. In addition to wake structure bifurcations at higher Re, the bifurcation scenarios at low Re without occurrence of the three characteristic instabilities are also studied.

2. Flow configuration and numerical method

In the present work, the flow is simulated using Large-Eddy Simulation (LES) approach. In LES, a low-pass spatial filtering operation is performed, i.e., only large-scale structures of the flow field are resolved and the small scales need to be modeled. The non-dimensional filtered continuity and momentum equations for the incompressible flow are (Pope, 2002):

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{\partial \bar{P}}{\partial x_i} + \frac{1}{\mathrm{Re}} \frac{\partial}{\partial x_j} \frac{\partial \bar{u}_i}{\partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} \tag{2}$$

where $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$ is the subgrid scale stress (SGS-stress), and the overbar indicates the spatial filtering.

The SGS-stress appears in an unclosed form and needs to be modeled. The dynamic Smagorinsky model (Germano et al., 1991) is used here:

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -2\nu_{SGS}\bar{S}_{ij} \tag{3}$$

where

$$\nu_{SGS} = (C_s \Delta)^2 |\bar{S}| \tag{4}$$

$$\bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right) \tag{5}$$

and

$$|\bar{S}| = \left(2\bar{S}_{ij}\bar{S}_{ij}\right)^{1/2} \tag{6}$$

ν_{SGS} is the subgrid scale viscosity, $\bar{S}_{ij}$ is the filtered rate-of-strain tensor, Δ is the filter width, and C_s is the Smagorinsky parameter. Here, Δ is equal to the size of the LES cell, and C_s is calculated dynamically using a scale similarity approach (Lilly, 1992).

A staggered grid system is used here to improve the difference scheme accuracy and numerical solution stability. The spatial discretization of the diffusive terms is based on a second-order central difference. The convective terms are discretized using a conservative second-order central difference scheme (Morinish et al., 1998), and the time derivatives are discretized with a second-order semi-implicit scheme. The pressure-correction equation is solved using the SIMPLE algorithm.

The computational domain consists of a cylinder of radius $Rc = 10d$ and length $L = 20d$, as shown in Fig. 1. The mesh extends $5d$ upstream and $15d$ downstream of the disk, to allow sufficient