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High-order simplified thermal lattice Boltzmann method for incompressible thermal flows

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ABSTRACT

A high-order simplified thermal lattice Boltzmann method (HSTLBM) is developed in this paper for accurate and efficient simulation of incompressible thermal flows. The derivation of HSTLBM stems from the recently developed simplified thermal lattice Boltzmann method (STLBM) and incorporates high-order interpolation algorithms, which reflects an effective combination of local second-order reconstruction and global high-order scheme. By introducing virtual streaming nodes, HSTLBM decouples the streaming distance from the mesh spacing and then correlates them through high-order interpolation scheme. Delicate parametric studies indicate that adopting 5-point Lagrange interpolation and setting the streaming distance as 0.2 times of the mesh spacing could give optimal results which balances computational accuracy, stability, and efficiency well; and third-order of global accuracy can be achieved. HSTLBM inherits various merits of STLBM, especially its nice numerical stability. As a result, HSTLBM can give accurate and stable solutions on coarser meshes for problems at high Reynolds/Rayleigh numbers. Higher efficiency and lower memory cost can thus be expected. A series of benchmark tests are provided for comprehensive evaluation of HSTLBM in modelling two- and three-dimensional problems and on uniform/non-uniform meshes.

1. Introduction

Accurate and efficient simulation of thermal flows has always been an attractive topic in Computational Fluid Dynamics (CFD) community. Various numerical methods have been proposed in the past decades [1]. Among them, the lattice Boltzmann method (LBM), which is constructed on a mesoscopic scale, is welcomed by researchers due to its simplicity, kinetic nature, and explicitness [2–5]. Typical thermal models developed in the framework of LBM include the multispeed model [6,7], the passive scalar method [8,9], and the thermal energy distribution model [10–12].

Apart from its merits, several drawbacks of the thermal lattice Boltzmann method (TLBM) were uncovered in its development and application. Most significant ones include high cost in virtual memory, inconvenient implementation of physical boundary condition, lattice uniformity and poor numerical stability at high Reynolds/Rayleigh numbers [13–16]. Continuous efforts have been made to suppress these drawbacks. Representative ones include multiple-relaxation-time (MRT) LBM [3,17], interpolation supplemented LBM (ISLBM) [18], artificial compressibility method [19], entropic LBGK method [20], cascaded lattice Boltzmann method [21] etc.

The recently developed simplified thermal lattice Boltzmann method (STLBM), on the other hand, explores an alternative approach of evolution within the LBM framework, which could eliminate or alleviate the above drawbacks [22–24]. The evolution process in STLBM is essentially reconstructing solutions to the macroscopic equations recovered from TLBM and resolved in a predictor-corrector step. The relationship between the non-equilibrium part and the equilibrium part of the distribution functions, which is given by the Chapman-Enskog (C-E) expansion analysis, plays an imperative role in the reconstruction process, and is approximated in a manner with the second-order of accuracy in STLBM. The resultant formulations of STLBM are basically functions of equilibrium distribution functions. Since equilibrium distribution functions can be explicitly calculated from the macroscopic variables, STLBM directly updates the macroscopic variables instead of distribution functions. This greatly lowers the cost in virtual memory and facilitates implementation of physical boundary conditions. Moreover, numerical tests revealed that STLBM is a highly stable method which can maintain numerical stability at high Reynolds/Rayleigh numbers and on very coarse meshes [22,23].

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However, it is noted that contradiction exists between the good numerical stability and the second-order of spatial accuracy. Although STLBM can give stable results at high Reynolds/Rayleigh numbers, these results are inaccurate, which lowers its value of reference to real engineering problems. Therefore, reducing the numerical error, or increasing the numerical accuracy, is an attractive topic in nurturing the method, which is the motivation of the present research.

The numerical error, which is defined as $O(h^n)$, is affected by two major factors: the mesh spacing h and the order of accuracy n. To reduce numerical error, one could either refine the mesh size or increase the order of accuracy of the numerical model. Refining mesh size is the simplest solution, but greatly increases numerical efforts. The second strategy could give more accurate results on relatively coarser meshes, but somehow complicates the formulations and reduces the flexibility in boundary treatment and on non-uniform mesh. The conventional strategy of constructing high-order scheme of LBM or TLBM is quite tedious. If starting from the fundamental formulations of LBM, one has to improve the accuracy in the discretization of the velocity space, the streaming term and the spatial derivatives simultaneously [25–28]. Through this approach, more expansion terms emerge in the final formulations, and the explicitness of the method is usually sacrificed. Moreover, the flexibility of constructed high-order scheme on non-uniform and body-fitted meshes is another concern when adopting high-order schemes.

In this paper, we propose an alternative way to construct high-order simplified thermal lattice Boltzmann method (HSTLBM), which could combine advantages of the above two approaches of reducing numerical error. The principal notion of HSTLBM is an effective combination of local second-order reconstruction and global high-order scheme. By introducing virtual streaming nodes, HSTLBM decouples the streaming distance δ_x from the mesh spacing h, and then correlates them through high-order interpolation algorithms. The local reconstruction of flow variables is still in the second-order of accuracy in terms of δ_x. However, higher order of global accuracy can be achieved if h is much larger than δ_x. Apparently, decoupling and correlation are two key issues in developing HSTLBM, which corresponds to two important parameters: the ratio of δ_x to h ($\xi = \delta_x/h$) and the number of interpolation points (m). Selection of these parameters is made from the balance among numerical accuracy, stability, and efficiency. A parametric optimization study is performed in this paper to show that, when Lagrange interpolation algorithm is adopted, setting $\xi = 0.2$ and $m = 5$ gives the optimal results, and globally the third-order of accuracy can be achieved.

There was a concern that adopting interpolation algorithm could lead to high numerical dissipation in LBM [29]. It is noteworthy that the interpolation algorithm implemented in previous LB models is usually in the second-order of accuracy which is consistent with the order of accuracy of LBE. In the present HSTLBM, high-order interpolation is adopted. Specifically, 5-point Lagrange interpolation is utilized along each direction, which yields a truncated error in the fourth order $O(h^4)$. The numerical diffusion introduced by interpolation is thus reduced by two orders in theory. As a result, HSTLBM could effectively preserve accuracy when simulating high Reynolds/Rayleigh number flows on nonuniform meshes.

Since the proposed HSTLBM maintains the ground structure of STLBM, most merits of STLBM are inherited. Simplicity and explicitness are preserved. HSTLBM still reflects a direct evolution of macroscopic variables instead of the distribution function, which reduces the memory cost and facilitates implementation of high-order boundary condition of physical properties. The most intriguing characteristic of STLBM, i.e. the nice numerical stability, is also inherited by HSTLBM, which will be reflected in numerical tests at high Rayleigh numbers and on coarse meshes. Together with its high-order essence, HSTLBM can give converged and accurate results on coarse meshes, which makes it an ideal solution to real applications in high Reynolds/Rayleigh numbers. In addition, HSTLBM is highly flexible in dealing with non-uniform and body-fitted mesh, which makes it more competitive than other high-order schemes within LBM framework.

The organization of the remaining parts of the paper is: Section 2 gives a brief review of TLBM and STLBM; Section 3 presents the derivation of high-order simplified thermal lattice Boltzmann method (HSTLBM) as well as the implementation of high-order boundary conditions. A parametric optimization study is carried out in Section 4 to determine two key parameters in HSTLBM. Four typical benchmark tests are then presented in Section 5 for comprehensive evaluation of the robustness of HSTLBM as well as its flexibility on non-uniform and body-fitted meshes. Concluding remarks are finally made in Section 6.

2. Brief outline of simplified thermal lattice Boltzmann method

2.1. Thermal lattice Boltzmann method (TLBM) and Chapman-Enskog (C-E) expansion analysis

As a mesoscopic method, the thermal lattice Boltzmann method (TLBM) updates the distribution functions instead of the macroscopic variables. One popular TLBM model is the thermal energy distribution model which incorporates the thermal effect by introducing the internal energy distribution function [11,12]. The evolution of the thermal energy distribution model can be written as

$$f_\alpha(\mathbf{r}+\mathbf{e}_\alpha\delta_t, t+\delta_t) = f_\alpha(\mathbf{r},t) + \frac{f_\alpha^{eq}(\mathbf{r},t) - f_\alpha(\mathbf{r},t)}{\tau_\upsilon}, \alpha = 0,1,\ \ldots\ ,M \tag{1}$$

$$g_\alpha(\mathbf{r}+\mathbf{e}_\alpha\delta_t,\ t+\delta_t) = g_\alpha(\mathbf{r},\ t) + \frac{g_\alpha^{eq}(\mathbf{r},\ t) - g_\alpha(\mathbf{r},\ t)}{\tau_c},\ \alpha = 0,1,\ \ldots\ ,N \tag{2}$$

where f_α and g_α represent the density and the internal energy distribution functions along α direction, respectively; τ_υ and τ_c are single relaxation parameters which respectively correspond to the kinematic viscosity and the thermal diffusivity; δ_t is the time step applied in the model; M and N are the number of the lattice velocities in lattice velocity model. Quantities superscripted by "eq" denote the corresponding equilibrium state, which, by omitting minor terms in $O(\mathrm{Ma}^3)$, can be expressed as

$$f_\alpha^{eq} = \rho\omega_\alpha\left[1 + \frac{\mathbf{e}_\alpha\cdot\mathbf{u}}{c_s^2} + \frac{(\mathbf{e}_\alpha\cdot\mathbf{u})^2 - (c_s|\mathbf{u}|)^2}{2c_s^4}\right] \tag{3}$$

$$g_\alpha^{eq}(\mathbf{r},t) = \begin{cases} -\frac{2\rho e}{3}\frac{|\mathbf{u}|^2}{c^2} & \alpha = 0 \\ \frac{\rho e}{9}\left[\frac{3}{2} + \frac{3}{2}\frac{\mathbf{e}_\alpha\cdot\mathbf{u}}{c^2} + \frac{9}{2}\frac{(\mathbf{e}_\alpha\cdot\mathbf{u})^2}{c^4} - \frac{3}{2}\frac{|\mathbf{u}|^2}{c^2}\right] & \alpha = 1-4 \\ \frac{\rho e}{36}\left[3 + 6\cdot\frac{\mathbf{e}_\alpha\cdot\mathbf{u}}{c^2} + \frac{9}{2}\frac{(\mathbf{e}_\alpha\cdot\mathbf{u})^2}{c^4} - \frac{3}{2}\frac{|\mathbf{u}|^2}{c^2}\right] & \alpha = 5-8 \end{cases} \tag{4}$$

where ρ, $\mathbf{u}$ and e are density, velocity vector, and internal energy, respectively; $c = \delta_x/\delta_t$ is equal to 1; δ_x is the lattice spacing; and the weighting coefficients ω_α, the sound speed c_s and the lattice velocity directions $\mathbf{e}_\alpha$ are

$$\omega_0 = \frac{4}{9} \quad \omega_{1-4} = \frac{1}{9} \quad \omega_{5-8} = \frac{1}{36} \quad c_s = \frac{c}{\sqrt{3}} \tag{5}$$