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Accelerated laminar flow near the heated permeable surface

A.Yu. Sakhnov

Kutateladze Institute of Thermophysics SB RAS, 1 Lavrentyev Ave., Novosibirsk, Russia

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ABSTRACT

The paper reports on numerical simulation of an accelerated laminar boundary layer over the heated permeable wall. Xenon was used as a working gas. The finite difference method was applied to solve the system of laminar boundary layer equations with variable gas properties. It has been shown that the gas injection in the mentioned conditions increases maximal velocity within the boundary layer and decreases skin-friction coefficient and thermal Stanton number. On the contrary, gas suction decreases an overshoot phenomenon down to its vanishing and increases skin-friction coefficient and thermal Stanton number up to the value at asymptotic suction. The paper considers a separation of thermal boundary layer at gas injection. The proposed conventional point of this separation is situated farther from the leading edge than the separation point of the dynamic boundary layer.

1. Introduction

Various technical systems use flow acceleration and gas injection or suction through the permeable wall. As it is shown in paper [1] an application of the convergent channel in the evaporator of refrigerator increases heat transfer coefficient and thereby allows decreasing the size of the evaporator. Usually, gas suction is used to stabilize the boundary layer [2,3]. However, Zammert et al. [4] showed that the wall heating disturbs the flow even in the presence of asymptotic suction. In paper [5] authors used Large Eddy Simulation to show that suction increases skin-friction coefficient significantly and gas injection decreases it. In experimental works [6,7] Kays et al. demonstrated that at the asymptotic suction Stanton number and skin-friction coefficient equal the magnitude of suction. Following Kays and Moffat [8], we use the definition of asymptotic boundary layer as the flow with "a state of equilibrium for which Re^{**} is constant".

Gas injection is used to control the boundary layer. Kornilov [9] showed that the gas injection can decrease the skin-friction coefficient to 90 per cents of this parameter for the impermeable surface. Gas injection is often applied for cooling the near-wall area of the hot flow both in a certain local part of the wall [10] and over the whole wall [11]. At that, in study [11] the gas is injected into the flow with favorable pressure gradient. There are also cases of hot gas injection into the cold main stream. Mirhoseini and Boroomand [12] studied local injection of the hot water vapor into the steam flow with the lower temperature in order to decrease the liquid fraction at the turbine output.

Kumari and Nath [13] obtained an overshoot phenomenon in the compressible flow over the heated permeable rotated sphere. This phenomenon is that the velocity within the boundary layer exceeds the main flow velocity. Authors demonstrated that the gas injection significantly increases the maximal velocity within the boundary layer.

Papers [14–16] contain investigations of the mixed convection flow around the heated permeable wedge. Authors obtained the overshoot phenomenon and showed that the gas injection increases the maximal velocity within the boundary layer and the gas suction, on the contrary, decreases it to total disappearance of the velocity maximum. Results of papers [15,16] show that in the specified conditions the gas injection decreases the skin-friction coefficient and Nusselt number, and the gas suction, on the contrary, increases friction and heat transfer in the boundary layer.

The flow with the overshoot within the boundary layer is sufficiently close by its shape to the near-wall jet. Amitay and Cohen [17] showed that gas injection and suction, respectively, increase and decrease the velocity maximum in the near-wall jet. At that, the gas injection moves the jet away from the wall and gas suction brings it to the surface.

The overshoot phenomenon arises in different near-wall flows such as wall heating, heterogeneous injection and chemical reactions [18]. In paper [19] we demonstrated that the indispensable factors of the overshoot existence are the favorable pressure gradient and the difference in densities of the near-wall area and the

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E-mail address: aleksei_sakhnov@mail.ru

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Nomenclature

c_f	skin-friction coefficient
c_p	gas specific heat at constant pressure [J/(kg deg)]
$\overline{j_w} = \rho_w V_w / \rho_e U_e$	dimensionless gas injection/suction intensity
$K = (\mu_e / \rho_e U_e^2) dU_e/dx$	stream acceleration parameter
L	length of wall [m]
P	pressure [Pa]
Pr	Prandtl number
Re_x	Reynolds number, based on streamwise coordinate x
$\mathrm{Re}_{\mathrm{int}}$	integral Reynolds number
Re_{x0}	Reynolds number, based on fixed initial velocity U_0
Re^{**}	momentum thickness Reynolds number
$\mathrm{St}_T = q_w / \rho_e U_e c_p \Delta T$	thermal Stanton number
T	temperature [K]
U, V	velocity components in the x, y directions respectively [m/s]
x, y	streamwise and normal coordinates relative to surface of streamlined body [m]

Greek symbols

α	the convergence angle of top plane of channel [degrees]
δ	thickness of hydrodynamic boundary layer [m], $U/U_e = 0.995$ or 1.005
$\delta^* = \int_0^\infty (1 - \rho U / \rho_e U_e) dy$	displacement thickness [m]
$\delta^{**} = \int_0^\infty \rho U / \rho_e U_e (1 - U/U_e) dy$	momentum thickness [m]
δ_T	thickness of thermal boundary layer [m], $\Theta = 0.995$
$\delta_T^{**} = \int_0^\infty \rho U / \rho_e U_e (1 - \Theta) dy$	enthalpy thickness [m]
λ	heat conductivity [W/(m · K)]
μ	dynamic viscosity [Pa · s]
$\Theta = (T - T_w)/(T_e - T_w)$	dimensionless temperature
ρ	density [kg/m^3]
$\psi = T_w / T_e$	temperature ratio
Ψ_{aw}	relative asymptotic skin-friction function

Subscripts

0	flow quantities at the start of flow
e	flow quantities in external flow
max	flow quantities at the point of maximum velocity
w	parameter at the wall

main flow ($\rho_w < \rho_e$). A short review of various flows with the overshoot phenomenon is given in the introduction of that paper. In [20] we obtained that effects of Prandtl number, viscosity, heat conductivity and heat capacity on the overshoot are secondary relative to the effect of the gas density.

The work [21] presents the results of numerical and analytical investigation of the laminar isothermal boundary layer with favorable pressure gradient over the permeable surface. The analytical solution of the boundary layer equations was obtained for the asymptotic flow conditions. Based on this solution we suggested the relative asymptotic skin-friction function Ψ_{aw}, which determines the degree of influence of flow acceleration and permeable wall on the flow. There are ranges of this function, where effects of permeable wall and streamwise pressure gradient have to be considered only in combination. Numerical simulation has shown that such combined influence of favorable pressure gradient and permeable wall extends the asymptotic flow. The study of strong gas injection into the accelerated flow has revealed that favorable pressure gradient impedes the boundary layer separation. At that asymptotic flow starts from the point, where the separation would occur at gas injection into the zero-pressure gradient flow.

This paper presents the results of numerical modeling of the laminar boundary layer with favorable pressure gradient over the heated permeable wall. The study is a development of works [19,21] and, accordingly, has a dual goal. On the one hand, we planned to show the influence of gas injection and suction on the overshoot phenomenon in the forced convection flow. On the other hand, the effect of the wall heating on the asymptotic flow regime in the accelerated boundary layer over the permeable wall will be studied.

2. Flow configuration and modeling approach

We consider xenon flow in a plane convergent channel where the acceleration parameter $K = (\mu_e / \rho_e U_e^2) dU_e/dx$ remains constant over the entire channel length (Fig. 1). The lower permeable wall is heated at a constant temperature T_w, higher than the main stream temperature T_e. On the lower wall of the channel, we set the gas suction or homogeneous injection with constant intensity $\overline{j_w} = \rho_w V_w / \rho_e U_e = const$ along the flow direction. The sloped top wall of the channel was assumed to be located quite far from the lower channel wall, i.e. $h_0 >> \delta$, so that the analysis can be focused on the accelerated hydrodynamic and thermal boundary layers, developing over the bottom surface. All parameters across the boundary layer and dimensionless characteristics are based on main flow quantities designated by subscript “e”.

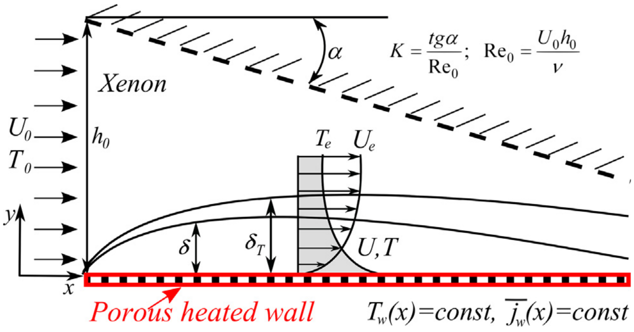

Fig. 1. Schematics of the flow considered.

2.1. Equations and boundary conditions

The flow considered can be well approximated by the parabolized two-dimensional momentum, continuity and energy equations for a steady laminar compressible boundary layer:

$$\rho U \frac{\partial U}{\partial x} + \rho V \frac{\partial U}{\partial y} = -\frac{dP}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial U}{\partial y}\right), \tag{1}$$

$$\frac{\partial(\rho U)}{\partial x} + \frac{\partial(\rho V)}{\partial y} = 0, \tag{2}$$

$$c_p \rho U \frac{\partial T}{\partial x} + c_p \rho V \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + U \frac{dP}{dx} + \mu \left(\frac{\partial U}{\partial y}\right)^2. \tag{3}$$

Xenon properties were calculated depending on temperature and pressure using the method described in paper [22]. Pressure equaled 0.1 MPa for all considered flow cases.