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Research on macroscopic and microscopic heat transfer mechanisms based on non-Fourier constitutive model

Lin Liu [a,*], Liancun Zheng [a], Fawang Liu [b]

[a] School of Mathematics and Physics, University of Science and Technology Beijing, Beijing 100083, China
[b] School of Mathematical Sciences, Queensland University of Technology, GPO Box 2434, Brisbane, Queensland 4001, Australia

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ABSTRACT

Based on the Cattaneo-Christov model and dual-phase-lag model, a novel constitutive model is proposed to study macroscopic and microscopic heat transfer mechanisms in the moving media. Formulated governing equation contains relaxation parameters and time fractional derivative with the highest order of $1+\alpha$ ($0<\alpha\leq 1$) which possesses both the relaxation characteristic and memory characteristic. With the help of L1-scheme, solutions are obtained by numerical difference method. Two applications with the new proposed model are given. One is to analyze the heat conduction in processed meat which analyzes the effects of convection velocity on the temperature distribution. The other is to study the thermal behavior of a biological tissue that the spatial evolution of the temperature distribution with the effects of involved parameters, such as fractional parameter, convection velocity parameter, macroscopic relaxation parameter and microscopic one is spotlighted by graphical illustrations. An interesting result is found that the temperature transports faster at the smaller x while slower at the larger x for a larger fractional parameter α, a smaller β or a larger microscopic relaxation parameter. For a larger macroscopic relaxation parameter or convection velocity parameter, the temperature transports slower.

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1. Introduction

The study of heat conduction is an important research field for its extensive application in widespread fields. The classical constitutive model to study heat conduction is based on the Fourier's law [1] which describes the linear relationship between the heat flux vector $\mathbf{q}$ and temperature gradient, given as:

$$\mathbf{q} = -\kappa \mathrm{grad} T, \tag{1}$$

where κ refers to the thermal conductivity.

The Fourier's law is adopted as a basis model for analyzing the heat conduction phenomenon. With the development of science and technology, the validity has been questioned due to the fact that it corresponds to an infinite propagation velocity [2,3]. Many modifications of the Fourier model have been proposed [4–7]. One of the classical modified constitutive models is proposed by Cattaneo [8] by introducing the macroscopic relaxation parameter ξ_q, given as:

$$\mathbf{q} + \xi_q \frac{\partial \mathbf{q}}{\partial t} = -\kappa \mathrm{grad} T, \tag{2}$$

here, the propagation velocity becomes $v=\sqrt{\kappa/\xi_q}$ and it reduces to the classical model (1) when $\xi_q \to 0$.

Based on the Cattaneo model, many researchers have given further improvements. One is called the Cattaneo-Christov model [9] which is proposed by substituting the time derivative for the upper-convection derivative, given as:

$$\mathbf{q} + \xi_q \left[\frac{\partial \mathbf{q}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{q} - \mathbf{q}\cdot\nabla\mathbf{V} + (\nabla\cdot\mathbf{V})\mathbf{q}\right] = -\kappa \mathrm{grad} T, \tag{3}$$

where $\mathbf{V}$ denotes the convection velocity vector of the material point.

The motivation of this generalized model is that the Cattaneo model has been verified not suitable for the moving media by experiment [10]. Many researchers have been devoted to study the effectiveness of this modified constitutive equation. Han et al. [11] applied the Cattaneo-Christov model to study the coupled flow and heat transfer in viscoelastic fluid above a stretching plate with velocity slip boundary. Later, Hayat et al. [12] analyzed the impact of Cattaneo-Christov heat flux model in flow of variable thermal conductivity fluid over a variable thicked surface and the results showed that the temperature profile decreases for higher thermal relaxation parameter. Liu et al. [13] proposed the space Riesz fractional Cattaneo-Christov model to characterize heat

* Corresponding author.
E-mail address: liulin@ustb.edu.cn (L. Liu).

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conduction phenomena and analyze the coexisting characteristics of parabolic and hyperbolic. More references related to the Cattaneo-Christov models can be seen in Refs. [14–16].

Besides, for the Cattaneo constitutive relationship, it only considers macroscopic relaxation parameter. The other modification to the Cattaneo model proposed by Tzou [17,18] is called the dual-phase-lag model, the expression is given as:

$$\mathbf{q} + \xi_q \frac{\partial \mathbf{q}}{\partial t} = -\kappa \mathrm{grad}\left(1 + \xi_T \frac{\partial}{\partial t}\right) T, \tag{4}$$

where ξ_T refers to the microscopic relaxation parameter.

The motivation of this modified model is to incorporate the microscopic effects into the macroscopic description which claims to build smoothly bridge between the macroscopic and the microscopic approaches. This constitutive relationship covers a wide range of heat transfer fields and attracts a large number of researchers to study. Among the researchers, Antaki [19] offered a new interpretation for the experiments of Mitra et al. [20] by using the dual-phase-lag model. By applying the generalized dual-phase-lag model, Kałuza et al. [21] gave sensitivity analysis of temperature field in the heated soft tissue with respect to the perturbations of porosity. Shen and Zhang [22] investigated a number of notable physical anomalies concerning non-Fourier heat conduction under the dual-phase-lag model. A list of key references concerning the dual-phase-lag model is given in Refs. [23–25].

Motivated by the above discussions, the constitutive model which contains the macroscopic relaxation parameter to study convection heat transfer process and the constitutive model which contains both the macroscopic relaxation parameter and the microscopic one to study heat transfer process have been widely used and discussed. Building a constitutive model which contains the macroscopic and microscopic relaxation parameters to describe the heat transfer process in a moving media is necessary and important to be put on the agenda and the new proposed model is given as:

$$\mathbf{q} + \xi_q \left[\frac{\partial \mathbf{q}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{q} - \mathbf{q}\cdot\nabla\mathbf{V} + (\nabla\cdot\mathbf{V})\mathbf{q}\right] = -\kappa \mathrm{grad}\left(1 + \xi_T \frac{\partial}{\partial t}\right) T. \tag{5}$$

Besides, as the generalization of integer derivative, the fractional operator possesses the memory and nonlocal characteristics [26,27] which is dependent on the whole points while the integer derivative is only a local one which is dependent on its nearby points. With the development of scientific research, the superiority of fractional derivative has been verified. Bagley and Torvik [28] gave a theoretical basis for the application of fractional calculus to viscoelasticity, concluding that fractional calculus models of viscoelastic materials can be better consisted with the physical principles. Meral et al. [29] verified that the fractional order Voigt models performed better than the integer order models. Song and Jiang [30] studied on the constitutive equation with fractional derivative for the viscoelastic fluids, verifying that the measured dynamic moduli and the theoretical predictions with fractional Jeffreys model showed an excellent agreement. For more references about the applications of fractional operators, see in Refs. [31,32].

Based on the fractional Cattaneo-Christov model [33] and fractional dual-phase-lag model [34], by introducing the time fractional parameter, the constitutive model (5) changes as:

$$\mathbf{q} + \xi_q \left[\frac{\partial^\alpha \mathbf{q}}{\partial t^\alpha} + \mathbf{V}\cdot\nabla\mathbf{q} - \mathbf{q}\cdot\nabla\mathbf{V} + (\nabla\cdot\mathbf{V})\mathbf{q}\right] = -\kappa \mathrm{grad}\left(1 + \xi_T \frac{\partial^\alpha}{\partial t^\alpha}\right) T, \tag{6}$$

where the symbol $\frac{\partial^\alpha}{\partial t^\alpha}$ refers to the Caputo derivative of order α $(0 < \alpha \leqslant 1)$, the definition [35–37] is given as:

$$\frac{\partial^\alpha T(x,t)}{\partial t^\alpha} = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{1}{(t-\tau)^\alpha} \frac{\partial T(x,\tau)}{\partial \tau} d\tau, \tag{7}$$

here, $\Gamma(\cdot)$ represents the Euler gamma function.

The fractional Cattaneo-Christov model [33] and the fractional dual-phase-lag model [34] have been applied to study heat conduction. As a generalization of the two models, the constitutive model (6) is a novel one which has not been applied to study the convection heat transfer process. Inspired by the above analyses, we study in this paper the anomalous diffusion in comb model with a view to study the temperature distribution with the effects of different parameters where the new proposed constitutive model is applied. The paper is organized as follows. In Section 2, the modified governing equation with the fractional parameters, macroscopic and microscopic relaxation parameters and convection velocity is formulated. The numerical solution of the new formulated equation is obtained in Section 3. Section 4 mainly presents the comparison between the exact solution and the numerical solution which demonstrates the accuracy of numerical discretization scheme. In Section 5, two applications of the new proposed model are given, one is to analyze the heat conduction in processed meat and the other is to study the thermal behavior of a biological tissue that the temperature distribution is spotlighted graphically. The main conclusions are presented in Section 6.

2. Mathematical formulation

For the one-dimensional situation and constant convention velocity, the constitutive model (6) is simplified as:

$$q + \xi_q \left(\tau_1^{\alpha-1} \frac{\partial^\alpha q}{\partial t^\alpha} + u \frac{\partial q}{\partial x}\right) = -\kappa \left(1 + \xi_T \tau_1^{\beta-1} \frac{\partial^\beta}{\partial t^\beta}\right) \frac{\partial T}{\partial x}, \tag{8}$$

where u refers to the convection velocity along the x direction. Here, The dimension for the first item q is W/m^2 while the dimension for $\xi_q \frac{\partial^\alpha q}{\partial t^\alpha}$ is W s$^{1-\alpha}$/m^2. In other words, the introduction of fractional parameter makes the dimension not conserve. Thus, the parameter τ_1 with dimension "s" is introduced to keep the dimensions in order.

Combining (8) with the conservation law of energy:

$$c\rho \frac{\partial T}{\partial t} + c\rho u \frac{\partial T}{\partial x} + \mathrm{div}\ q + f_1(x,t) = 0, \tag{9}$$

we obtain the final governing equation:

$$\xi_q \tau_1^{\alpha-1} D_t^{\alpha+1} T + \xi_q u \tau_1^{\alpha-1} D_t^\alpha \frac{\partial T}{\partial x} + \xi_q u \frac{\partial^2 T}{\partial x \partial t} + \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + \xi_q u^2 \frac{\partial^2 T}{\partial x^2} - a \frac{\partial^2 T}{\partial x^2} - a \xi_T \tau_1^{\beta-1} D_t^\beta \frac{\partial^2 T}{\partial x^2} + f(x,t) = 0 \tag{10}$$

with dimensionless initial conditions:

$$T(x,0) = \phi_1(x), \quad \frac{\partial T(x,0)}{\partial t} = \phi_2(x), \tag{11}$$

and boundary conditions:

$$T(d_1,t) = \varphi_1(t), \quad T(d_2,t) = \varphi_2(t), \tag{12}$$

where $a = \kappa/(c\rho)$ is the thermal diffusion coefficient, d_1 and d_2 denote the region along the x direction, $f_1(x,t)$ refers to a source item and $f(x,t) = \frac{1}{c\rho}\left[f_1(x,t) + \xi_q \tau_1^{\alpha-1} \frac{\partial^\alpha f_1(x,t)}{\partial t^\alpha} + \xi_q u \frac{\partial f_1(x,t)}{\partial x}\right]$. The physical meaning of symbol $f_1(x,t)$ is different for various research backgrounds.

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