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An analytical study of the periodic laminar forced convection of non-Newtonian nanofluid flow inside an elliptical duct

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ABSTRACT

In this study, an analytical solution was obtained for a laminar forced convection of non-Newtonian nanofluid flowing inside an elliptical duct, with inlet temperature varying periodically with the time. The solution was obtained using the Generalized Integral Transform Technique (GITT). The thermal behavior of Cu-water non-Newtonian nanofluid, described by the power-law model was investigated. Also, an accurate correlation was established to estimate the thermal length required to achieve 99% of the amplitude attenuation. The results show a significant effect of aspect ratio β/α and fluid behavior index on the temperature amplitude reduction. For instance, an elliptical duct with $\beta/\alpha = 0.25$ reduces the thermal length L_{th} more than 50% compared with circular duct. Adding nanoparticles until 5% increases the heat transfer coefficient up to 27% for the cylindrical tube. Besides, the heat transfer coefficient is improved over 42% relatively to the cylindrical configuration by reducing the aspect ratio to 0.25. Therefore, adding 5% of nanoparticles both using an elliptical duct with $\beta/\alpha = 0.25$, improves the mean heat transfer coefficient around 83% compared to the flow of water base fluid inside a cylindrical duct.

1. Introduction

Usually, heat exchangers are designed according to the inlet temperature and the desired outlet temperature of the working fluid. If the inlet temperature is subjected to a periodic variation, the thermal response of the heat exchanger can be highly distorted. To understand the heat exchange behavior where the fluid flowing inside the duct is subjected to temperature fluctuation, it is important to solve the Graetz problem under a periodic inlet temperature assumption [1,2]. Shortly called periodic Graetz problem, its solution has a great interest in engineering applications since it helps in predicting the thermal behavior of heat exchangers subjected to the unsteady thermal conditions. The results can be used also in building a more efficient heat stabilizers and heat rectifiers.

The first study of this problem was undertaken by Sparrow and Farias [3]. The authors analyzed analytically the heat transfer of slug flow between two parallel plates with periodic inlet temperature by considering wall conjugation effects. They applied the separation of variables method where the yielded Sturm-Liouville problem was solved by a trial and error procedure. Several difficulties were met in the evaluation of the complex eigenquantities. This is due to the fact that the solution of the periodic Graetz problem is defined in the complex space. Afterwards, numerous authors had applied different mathematical methods to overcome the inherent difficulties of the complex eigenquantities. Cotta and Ozisik [1] used the Generalized Integral Transform Technique (GITT) [4], to investigated the heat transfer of fully developed laminar fluid flow in circular duct and parallel plates, subjected both to a periodic inlet temperature and a constant wall temperature. Later, Cotta et al. [2] treated the case described in [3], by using a modified sign-count method. Kim and Ozisik [5] solved the same previous case, including the parabolic velocity profile. Their proposed solution was obtained through the use of shooting method. Travelho and Santos [6] solved the same problem by using the Laplace transform method. Guedes and Cotta [7] considered, in addition, the heat conduction in a thin wall and the solution is obtained using the GITT method. In another paper, Guedes and Ozisik [8] proposed a hybrid approach between the second-order finite differences and the GITT method to solve the periodic Graetz problem of laminar flow between two parallel plates. An approximate analytical solution of the last problem has been obtained by Fourcher and Mansouri [9] using the second-order Galerkin method. Ünsal [10,11] used the method of matched asymptotic expansion called Nayfeh's method [12], to solve the periodic Graetz problem in parallel plates [10] and circular pipe [11]. Cheroto at al. [13] proposed a solution based on integral transform of the periodic laminar forced convection problem in ducts using symbolic

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Nomenclature

a, b	ellipsis major and minor semi-axes' length (m)
A	dimensionless temperature amplitude
B, C, D, F	equation coefficients
[D], [F], [H]	matrix coefficients
c	integration constants
Cp	specific heat (J/kg.K)
D_h	hydraulic diameter (m)
f	frequency (Hz)
h	heat transfer coefficient (W/m^2.K)
i	complex number $i^2 = -1$
j, k, m, n	integer indices
J	Jacobian determinant
K	consistency factor of the power law model (Pa.s$^\eta$)
L	duct length (m)
L_{th}	dimensionless thermal length
$L_{th,\ real}$	thermal length (m)
M, N	number of expansion terms
$\dot{M}$, $\dot{N}$	normalization integrals
Nu	Nusselt number
P	perimeter of the ellipsis
Pe	Péclet number
r, s	integer matrix indices
Re_g	generalized Reynolds number for power law fluid
t	time variable
T	temperature (K)
T_0	inlet and wall temperature (K)
ΔT	amplitude of the inlet temperature fluctuation (K)
u, v	elliptical curvilinear coordinates
U	dimensionless velocity field
v_0	perimeter coordinate in (u,v) system
V	velocity field (m/s)
x, y, z	space coordinates (m)
X, Y, Z	dimensionless space coordinates

Greek symbols

α, β	dimensionless semi-axes' length
$\widehat{\alpha}$	dimensionless focal distance
γ, λ	eigenvalues
Γ	ellipsis contour
δ	ratio of the nano-layer thickness to the nanoparticles radius
η	flow behavior index of the power law model
θ	dimensionless temperature
κ	thermal conductivity (W/m.K)
Λ	ellipsis surface
μ	dynamic viscosity (Pa.s)
ξ, ψ	eigenfunctions
ρ	density (kg/m^3)
ρCp	volumetric heat capacity (J/m^3.K)
σ, χ	eigenvalues and eigenvectors of the matrix [H]
τ	dimensionless time variable
φ	nanoparticles volume fraction (%)
Φ, Ψ	normalized eigenfunctions
ϕ	phase lag (rad)
ω	frequency (rad/s)
Ω	dimensionless frequency

Subscripts

av	average
b	bulk
c	complex
f	base fluid
p	nanoparticle
nf	nanofluid

computation. Kakaç and Li [14] conducted an experimental and theoretical investigation of the turbulent forced convection in parallel plates with periodic inlet temperature. The wall thermal capacitance and the external convection were considered and the analytical solution was obtained using the GITT. The results were confronted to the experimental values and the comparisons showed a gratifying agreement. Santos et al. [15] and Cheroto et al. [16] investigated the laminar forced convection in simultaneously developing flow with periodic inlet temperature. The considered boundary condition was the fifth kind taking into account the external convection and the wall capacitance. The solution were obtained analytically by the GITT, however, the results and discussions were more extended in [16]. Cossali [17] worked out a solution as a series of Kummer functions for the Graetz problem with periodic inlet temperature in various shapes (sinusoidal, rectangular, etc.), with two kinds of wall boundary condition, namely constant temperature and constant heat flux. Hadiouche and Mansouri [18,19] solved the periodic Graetz problem with the fifth type of wall boundary condition by using two methods, the GITT in [18] and the variational method in [19]. They obtained a satisfactory agreement when the results were compared to those obtained through the use of quasi-steady approach that assumes a constant heat transfer coefficient at liquid–solid interface. Knupp et al. [20] reduced the physical scale of the periodic Graetz problem to the microscopic level. They analyzed the transient conjugated heat transfer in microchannel using the GITT with single domain formulation. In their assumptions, they considered the axial diffusion effects, in addition to the pre-cooling or the pre-heating of the wall.

From the studies reviewed above, to our knowledge, the periodic Graetz problem was only investigated for Newtonian fluid flowing inside a circular pipe, between parallel plates [1–3,5–11, 13–19] and recently in micochannels [20]. However, in practice, several types of ducts can be used in building up heat exchangers such as rectangular duct, triangular and elliptical duct or more complex configuration such as hexagonal and twisted duct where the working fluid is not necessary Newtonian. The main reason of the use of these pipes is to increase the overall heat transfer area which allows to reduce the volume and the weight of the heat exchangers. The elliptical duct fit perfectly in this context as for the same cross-section area, the elliptical duct has a widely larger perimeter than the circular duct. Studies showed that the use of elliptical duct enhances significantly the heat transfer rate [21–24]. In addition, from the practical view, most fluids (natural or synthetic) show a complex dynamic behavior and characterized as non-Newtonian fluids. Several studies showed that the thermal response of these fluids is highly influenced by the flow behavior [25–27].

Nanofluids are suspensions composed of a conventional liquid (such as water, ethylene glycol, etc.) usually called base fluid, and solid nanoparticles (Al_2O_3, CuO, TiO_2, etc.) where their sizes are in the scale of (1–100 nm). Depending on their shape, concentration and physical properties, adding nanoparticles to a base fluid enhances significantly the thermal conductivity of the resulted mixture [28,29]. It should be noted that many scholars [30–32] indicated that adding nanoparticles to the base fluid shows a shear-thinning effect (pseudoplastic behavior). Due to their low