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A coupled Volume Penalization-Thermal Lattice Boltzmann method for thermal flows

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ABSTRACT

In this article, a coupled Volume Penalization-Thermal Lattice Boltzmann method is proposed to solve the thermal flow problem. The temperature Dirichlet boundary condition of the temperature field is ensured by introducing an external thermal penalization heat source term into the energy equation. Coupled with the Lattice Boltzmann-Volume Penalization method, which is used to simulated athermal flow past obstacles, the thermal flow problem can be solved. Besides, performing the Volume Penalization-Thermal Lattice Boltzmann method on a certain point, only the variables of this point are needed, which means the present method can be conducted parallelly. To verify the present method, the heat transfer between two concentric circular cylinders experiment is carried out firstly, in which the accuracy of the present method is also studied. Then natural convection between two concentric circular cylinders and between a cold square outer and a hot cylinder inner is performed to verify the present method further. To validate the ability of the method to solve the forced convection and mixed convection, the flows past a heated circular cylinder and the mixed convection of a heated rotating cylinder in a square enclosure are conducted. Good agreements between the present results and those in the previous literatures are achieved.

1. Introduction

Lattice Boltzmann method, as an alternative to the traditional Navier-Stokes (N-S) equation solver, has been adopted to solve the problem relating to the interaction between fluid flows and obstacles widely [1]. The simplicity in coding, parallel and explicit calculation procedure are its three main advantages which contribute to its popularity. In the procedure of the solving the interaction between the fluid flows and the obstacles, especially when the boundary of the obstacles are complex, treating the obstacle boundaries is an extremely important point. Just like in the traditional N-S equation solvers, two main techniques: body-fitted grid method and the immersed boundary method are adopted to solve the interaction between the fluid flows and the obstacles in the Lattice Boltzmann method.

For the body-fitted grid method, generating a body-fitted grid is the first and important step, during which the structured and unstructured grids are frequently used. But this step is of great expense, especially when the some complex boundaries are involved. Even with simple boundaries, it is not easy to create a high quality body-fitted grid. When the boundary of the obstacle moves or changes, the body-fitted grid should be regenerated every step and the variables on the last grid should be also interpolated to the new grid, which is prohibitive. Besides, the order of accuracy on the structured and unstructured grids is lower than that on the uniform Cartesian grids [2].

Compared with the body-fitted grid method, the immersed boundary method, proposed by Peskin [3], can be easily implemented. In the immersed boundary method, an external forcing term is introduced to the momentum equations to reflect the boundary effect of obstacles on the fluid flows, which is the extremely bright spot of immersed boundary method. For the modified momentum equations, there is no inner boundary, which means the modified momentum equations can be solved on a fixed uniform Cartesian grid. As a result, the generation and re-generation of the body-fitted grid are unneeded, even when the boundary of the obstacle moves or changes. The dynamics of the obstacles are represented by a Lagrangian grid. The information and variables on these two grids are related to each other by a discrete delta function interpolation. Interpolating the velocity on the boundary based on the velocity field of the Cartesian grid and spreading the force density to the Cartesian grid points near the boundary by using the delta function are two main steps in the immersed

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boundary method. After the immersed boundary method was firstly incorporated into the Lattice Boltzmann method by Feng and Michaelides [4], many types of the coupled immersed boundary method and Lattice Boltzmann method have been developed for the incompressible viscous fluid flows past obstacles such as: the direct-forcing IBM-LBM [5], velocity correct-based IBM-LBM [6], momentum exchange IBM-LBM [7] and so on. But here another interesting immersed boundary method is focused: the Volume Penalization (VP) method.

The Volume Penalization method, proposed by Arquis and Caltagirone [8], was incorporated into LBM by Benamour [9]. In the VP, the obstacles are considered as a porous medium with extremely small permeability. The boundary of the obstacle is modeled on the fixed grid by a mask function. Actually, the Lagrangian grid in the VP is part of the fixed Cartesian grid which is marked by the mask function. So each point of the Lagrangian grid coincides with a certain point of the fixed Cartesian grid. Compared with the direct forcing IBM and the velocity correction IBM, there is no need to interpolate the velocity at the boundaries and to spread the force density to the Cartesian grid points near the boundaries by the delta function. So performing the VP procedure on a certain Lagrangian grid point just needs the variables on a Cartesian grid point with which the Lagrangian grid point coincides, which means the VP procedure can be conducted parallelly. Taking the parallelizability of the LBM into consideration, the whole VP-LBM can be conducted parallelly. Besides, the solution of the penalized N-S equations tends towards the exact solution of the N-S equations imposing no-slip boundary conditions with the penalization parameter approaching zero [10–12]. Under the help of the forcing term proposed by Guo [13], the VP is incorporated into LBM successfully to simulate flows past obstacles [14].

In this article, the volume penalization method coupled with Lattice Boltzmann is applied to solve the heat transfer problem. The heat transfer problem is a very interesting issue which has attracted many researchers and has been used in many areas, such as the latest reported research micropolar fluids [15], fluid flow in porous enclosure [16,17] and thermal flows under the control of electric field [18–20]. Coupled with the Thermal Lattice Boltzmann method (TLBM), the immersed boundary methods mentioned above have been also used to solve the heat transfer and convection problem [21–23]. Besides, M. Sheikholeslami and his co-workers have adopted the Thermal Lattice Boltzmann to study the natural convection heat transfer problems under the magnetic field [24], which has opened a new area where the Thermal Lattice Boltzmann can be used. An external thermal penalization heat source term is introduced into the energy equation to enforce the temperature field at the boundaries satisfied the temperature Dirichlet boundary condition. The modified energy equation is solved by an another population of Lattice Boltzmann model: Thermal Lattice Boltzmann method. Coupled with modified N-S equation solved by the Lattice Boltzmann method, the proposed method Volume Penalization-Thermal Lattice Boltzmann (VP-TLBM) is used to solve the thermal flow problem. To verify the proposed VP-TLBM and study the accuracy of the proposed method, the heat transfer between two concentric circular cylinders is conducted firstly. Then the natural convection between two concentrically placed horizontal circular cylinder and between a cold square outer and a hot circular cylinder inner is chosen as the experiment to verify the proposed method further.

The rest of this article is arranged as follows. In Section 2, the Lattice Boltzmann-Volume Penalization method firstly introduced briefly. Then the proposed VP-TLBM is introduced in details. The whole computational procedure is given in this section as well. In Section 3, the numerical experiments and the comparison of the results are given. Some concluding remarks and recommendations for the future work are presented in Section 4.

2. Mathematical and numerical formulation

In this section, the Lattice Boltzmann-Volume Penalization is introduced briefly. Then the proposed Volume Penalization-Thermal Lattice Boltzmann method is introduced in details. Finally, the whole computing procedure is given.

2.1. The Lattice Boltzmann-Volume Penalization method

Let us take the fluid-solid interaction (FSI) between incompressible viscous fluid and rigid boundary into consideration. The dynamics of the fluid can be governed by the following incompressible Navier-Stokes equation:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \frac{1}{\rho} \nabla p = \mu \nabla^2 \mathbf{u} + \mathbf{f} \tag{1}$$

$$\nabla \cdot \mathbf{u} = 0, \tag{2}$$

where $\mathbf{u}$ is the velocity of the fluid, μ is the dynamic viscosity, ρ is the density, p is the pressure and $\mathbf{f}$ is the body force. The no-slip boundary conditions on the rigid boundary domain Ω_O in the fluids can be described as:

$$\mathbf{u}|_{\partial\Omega_O} = \mathbf{U}_O, \tag{3}$$

where $\partial\Omega_O$ is the boundary of the obstacles and $\mathbf{U}_O$ is the velocity of the obstacles. The computational domain is shown in Fig. 1. Ω_F is the fluid domain. The union of these two domains $\Omega = \Omega_F \cup \Omega_O$ is the entire domain.

The Dirichlet problem Eqs. (1)–(3) can be solved by the Volume Penalization method [10,11]. In the Volume Penalization method, the solid obstacles are modeled as porous media. By adding a penalization term on the velocity, the momentum Eq. (1) is modified as:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f} - \frac{\chi(\mathbf{x}, t)\rho}{\eta}(\mathbf{u} - \mathbf{U}_O), \tag{4}$$

where

$$\chi(\mathbf{x}, t) = \begin{cases} 1 & x \in \Omega_O \\ 0 & other \end{cases} \tag{5}$$

is the mask function used to describe the obstacles' geometry and $\eta \ll 1$ is the penalization parameter. It can be seen that there is no Dirichlet boundary condition in Eq. (4). The solution of the

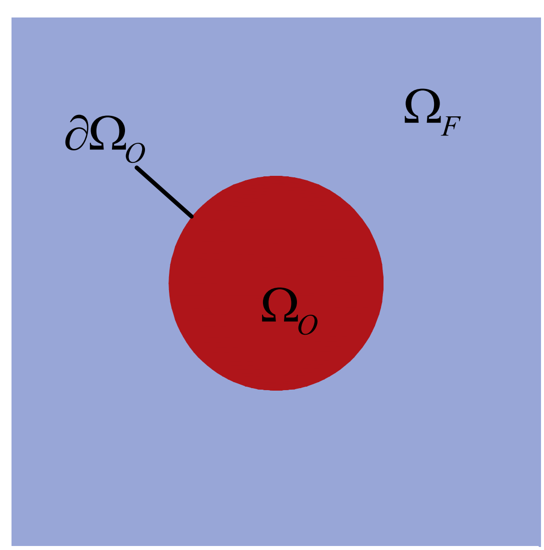

Fig. 1. The computational domain of boundary and fluid.