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Study of natural convection in a heated cavity with magnetic fields normal to the main circulation

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ABSTRACT

Both three-dimensional (3D) and quasi-two-dimensional (Q2D) numerical simulations are conducted to study the natural convection in an electrically insulated cavity with imposed horizontal magnetic fields (MF) normal to the main circulation, where two opposite vertical walls are kept at different temperatures and the other four are thermally insulated. We mainly focus on flows at Hartmann ($Ha \leqslant 10^4$) and Grashof ($Gr \leqslant 4 \times 10^7$) numbers. It is found that, with lower MF, the strengthening of the main convection, which is due to the suppression of the secondary flow along the MF lines by the Lorentz forces, results in the enhancement of heat transfer. Once the Q2D is achieved, the convection and the heat transfer would be damped by the further increase of MF. Our numerical results have confirmed that the damping velocity and kinetic energy of main convection are scaled as Gr/Ha (see Ref. Tagawa et al. (2002)). At the same time, the computed average Nusselt number (Nu) agrees well with the experimental results of Okada and Ozoe (1992), which can be correlated as a function of $Ha/Gr^{3/4}$ rather than $Ha/Gr^{1/3}$.

1. Introduction

In fusion reactor blankets, the studies of liquid metal with the MF have been conducted for a long time. Previously, more concerns have been paid on the issues of reducing the magnetohydrodynamic (MHD) pressure drop or inter-duct electromagnetic coupling effect [1], while the influence of thermal convection is rarely covered. In fact, the liquid metal, for removing the tritium produced within the fluid volume by the breeding reaction, flows at a very low velocity (a few mm/s) [2], so that the buoyant magnetoconvection appears very important. In particular, buoyancy-driven convection may cause some potential problems in blanket, such as safety and efficiency. The hot and cold spots or the large-amplitude low-frequency fluctuations of temperature [3–8] for example, resulting from the convection, may cause strong and unsteady thermal stresses in the walls, which are highly undesirable for DCLL [2] or HCLL [13]. Therefore, more and more laboratory experiments [3,4] and numerical simulations [5–12] on the buoyant magnetoconvection have been carried out in a wide range of Ha and Gr.

Several attempts of analysis of the thermal convection phenomena in various configurations have been performed with experiments, theories or numerical simulations. Ozoe and Okada [14] measured the heat transfer rates of natural convection of molten gallium under various strength of heating rates and magnetic fields with three different directions. The results revealed that cases with MF perpendicular to the main circulation appear less effective suppression than that in the other two directions. This has been confirmed by Tagawa [15–17] with 3D numerical simulations. Meanwhile, the enhancement of heat transfer rate by application of a perpendicular MF was found. The Hartmann layers convey all the electric current induced in the core and, therefore, limit the damping to velocities of the order of Gr/Ha, whereas in the other cases the velocity scales as Gr/Ha^2 (see Refs. [18,19]). However, no quantitative demonstration is available in their studies and the physical mechanism of transition from 3D to Q2D remains unexplored. Analogously, Garandet et al. [20], Alboussière et al. [21] and Davoust et al. [22] investigated buoyant flow in long horizontal enclosures with a various strength of magnetic field. When the MF is moderate, the flow organization with ascending and descending jets along the hot and cold walls, their possible separation from the walls and resulting formation of small cells, are quite specific to long vertical enclosures. In addition, simulations have been performed to study the effects of electrical conducted walls on the heat transfer rate combined with mechanism explanation [23,24]. Besides the situations mentioned above, the natural convection in a cubic combined with the influence of the horizontal or the inclined magnetic fields has been intensive studied by Chamkha et al. [25–27]. Through adjusting the thermal boundary conditions, such as walls with linearly heating [26] or heat source (sink) [25], and the kinematical boundary conditions, such as walls with a

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moving velocity or partial slip [25], flow and heat transfer have been investigated in detail.

Recently, more concerns have been given on investigating the effects of strong MF on conventional turbulence, such as Krasnov et al. [28] and Zikanov et al. [6,9,11,12]. The applied configuration is referred to the currently designed liquid metal blankets for future nuclear fusion reactors [29,2,30], where hypothetical flow is considered at a slow speed in a toroidal duct with volumetric internal heating. The thermal convection may cause turbulence at such a high *Gr*, which results in strong and nearly uniform heat transfer into the walls. However, due to the strong MF, the flow is nearly 2D with weak three-dimensional (3D) perturbations, even purely two-dimensional.

Here we propose to deal with the flow in a laterally heated three-dimensional box with electrically insulated walls in the presence of a strong horizontal MF normal to the initial temperature gradient Fig. 1. It is known that the presence of an applied uniform MF gives birth to induced electrical currents and Lorentz forces, which extend the flow structures along the MF lines [31]. If the imposed MF is sufficiently strong, the flow is invariant along the MF lines, except in the vicinity of the walls (normal to MF), called Hartmann layer, where viscous forces maintain strong velocity gradients Fig. 1(b). The resulting flow, composed of such Hartmann layers where the velocity grows exponentially from zero at the wall to the core value and a core flow where the velocity is constant along the MF lines, is called quasi-two-dimension (Q2D). With regard to this Q2D problem, Ref. [32] has developed such a model, hereafter SM82, by averaging the motion equations along the MF lines. This results in a 2D equation where Lorentz forces appear as a linear term due to the friction *they indirectly inducing by shaping the Hartmann layers* [33]. Neither empirical assumptions nor parameters are further required. Moreover, the numerical implementation of this model only involves the addition of the linear Hartmann damping to the classical 2D Navier-Stokes equation. The Q2D model used here has already been successful in predicting the mixed convection [5] and the linear stability for the Rayleigh-Benard problem [35], as well as for the isothermal mixing layer [34,36–38].

Therefore, with a weak MF, 3D numerical simulations are conducted because of the existence of 3D flow structures and enhancement of heat transfer in the present study. For the strong MF, the SM82 model has been applied to simulate the magneto-convection, due to the Q2D structural characteristic. In such way, the CPU costs can be reduced by a large extent. We focus on explaining the physical mechanism of enhancement of heat transfer rate in details and exploring the influence of MF on the velocity and heat transfer rate.

2. Problem statement and formulation

2.1. Governing equations

The configuration studied in the paper is a cavity filled with liquid metal whose Prandtl number ($Pr = \nu/\alpha$) is 0.025, as shown in Fig. 1(a). The applied MF is supposed to be perpendicular to the applied temperature gradient. As shown in Fig. 1(b), the middle horizontal $x-z$ cross-section is divided into three sub-regions: (i) core region; (ii) Hartmann layer whose thickness is l/Ha; (iii) Side layer, parallel to the MF, whose thickness is $l/Ha^{1/2}$, where l is the length of each side of the square cross-section. Fig. 1(c) shows the main circulation plane.

Here, the liquid metal is supposed to be a Newtonian fluid and the Boussinesq approximation is valid. The magnetic Reynolds number is very small, thus the induced MF can be negligible [39]. Moreover, compared with the heating from the walls, the contribution of the Joule dissipation can be neglected [31]. For such a fully three-dimensional flow, the non-dimensional equations are follows:

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \nabla \cdot (\boldsymbol{uu}) = -\nabla p + \nabla^2 \boldsymbol{u} + Ha^2(\boldsymbol{j} \times \boldsymbol{e}_B) + GrT\boldsymbol{e}_y \tag{2}$$

$$\boldsymbol{j} = -\nabla \Phi + \boldsymbol{u} \times \boldsymbol{e}_B \tag{3}$$

$$\nabla \cdot \boldsymbol{j} = 0 \tag{4}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T = \frac{1}{Pr} \nabla^2 T, \tag{5}$$

where the variables $\boldsymbol{j}, \Phi, \boldsymbol{u}, T$ denote the current density, electric potential, velocity and temperature, respectively.

Here, the following typical scales are used to non-dimensionalize the magnetohydrodynamic governing equations: l for length, ν/l for velocity, l^2/ν for time, $\rho\nu^2/l^2$ for pressure, the imposed field B_0 for MF, νB_0 for the electrical potential, $\sigma\nu B_0/l$ for electrical current density, and the temperature difference $\Delta T = T_h - T_c$, for temperature, where T_h represents the heated wall temperature and T_c is the cold wall temperature.

In MHD flows involving the heat transfer, the different cases considered can be characterized by three non-dimensional numbers. The first one is the Prandtl number, representing the ratio of viscous diffusivity to thermal diffusivity (α). The second one is the Grashof number, $Gr = g\beta(T_h - T_c)l^3/\nu^2$, representing the ratio of buoyant forces to viscous forces, of which β is the volumetric

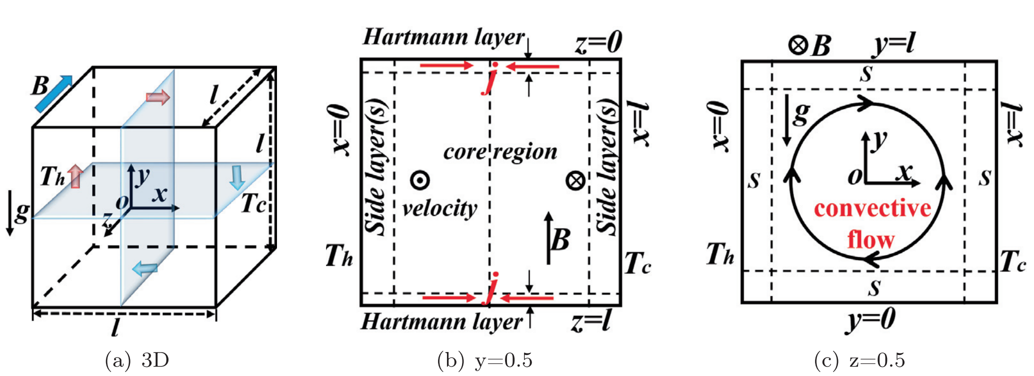

Fig. 1. (a) Schematic diagram of buoyant convection in a horizontal MF with the horizontal temperature gradient: a three-dimensional cavity; (b) middle horizontal cross-section divided into three sub-regions; (c) two-dimensional cavity for the averaged flow with respect to z and the position of the averaged Shercliff layers S of thickness O ($Ha^{1/2}$).