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Mixed convective three-dimensional flow of Williamson nanofluid subject to chemical reaction

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ABSTRACT

Main theme of this article is to model and analyze the outcome of chemically reactive flow of nanomaterial. Nanomaterial comprises thermophoresis and Brownian motion. Bidirectional nonlinear stretching sheet of constant thickness is considered. Rheological expressions of Williamson fluid is used to develop formulation. Boundary layer approach and suitable transformations are utilized to simplify the governing equations. Optimal homotopy analysis method (*OHAM*) is utilized for values of convergence control parameters. Tabulated values of skin friction coefficients and Nusselt and Sherwood numbers via different parameters are calculated and examined. Physical features of various pertinent parameters are argued through graphs. It is observed that velocity decays in x-direction for higher values of magnetic parameter. Temperature and concentration have contrast behavior for larger Brownian motion.

1. Introduction

Nanomaterials at present is continuous focus of the scientists and engineers. Nanomaterials are useful for increase in thermal efficacy of heat transport process. It is by the amalgamation of nano-sized particles into the base liquid. Pharmaceutical process, hybrid-powered engines, electronics cooling, solar collectors, engine and fuel cells and nuclear applications are examples of emerging nanotechnologies. Choi [1] verified that the amalgamation of nanoparticles in base liquid shows noteworthy increase in thermal conductivity of liquid. Currently the researchers employed nanomaterial as a way to improve heat transfer efficiency of fluids. Buongiorno [2] proposed the non-homogeneous equilibrium model which reveals that Brownian movement and thermophoretic diffusion of nanomaterials are the two noteworthy effects for the cause of sharp increase in heat transfer. Hayat et al. [3,4] studied the magnetic effect in second grade nanofluid flow by a stretching sheet. Hayat et al. [5] investigated stratified mixed convection thixotropic nanofluid flow by stretching surface. Farooq et al. [6] studied MHD viscoelastic nanoliquid flow subject to non-linear radiation. Representative analyses on the nanofluid flows can be seen in Refs. [7–10].

Due to numerous applications, the non-Newtonian fluids are used as a base fluid in many industrial processes. These include food mixing, plasma and lubrications with greases and oils. Most commonly used non-Newtonian fluids are pseudoplastic fluids. Boundary layer flow of such fluids is of much attention due to their vast applications in industry. Navier-Stokes equations are not sufficient to understand and elaborate the rheological properties independently. Therefore to overcome this deficiency, the researchers have proposed many rheological models like Carreau model, Ellis model, Cross model and Eyring Powell model. In this category, the Williamson fluid model is given less attention. Williamson [11] recommended a rheological model to study the pseudoplastic fluids flow and verified it experimentally. Nadeem et al. [12] elaborated the Williamson fluid flow by a stretching sheet. Noteworthy reviews on Williamson fluid are shown by Cramer et al. [13], Krishnamurty et al. [14], Hayat et al. [15–17], Salahuddin et al. [18] and Malik et al. [19].

Flow over a stretching surface is of central importance due to its various engineering applications in fluid film condensation, extrusion of polymer sheets from die and emulsion coating, paper production etc. Fang et al. [20]explored flow by a stretching sheet with varying thickness. Waqas et al. [21] studied MHD mixed convective flow past a nonlinear stretched sheet. Hayat et al. [22] discussed mixed convective flow of viscous liquid with chemical reaction between two rotating disks. Flow of third grade liquid by an exponentially stretching surface with chemical reaction

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and magnetohydrodynamics is examined by Hayat et al. [23]. Farooq et al. [24] explored MHD stagnation point flow of non-Newtonian liquid with double stratified medium towards nonlinear stretchable sheet. Hayat et al. [25] scrutinized Darcy-Forchheimer flow of viscous liquid with nonlinear radiation and homogeneous-heterogeneous reactions. Hayat et al. [26] studied flow of viscoelastic liquid by a nonlinear radially stretching sheet with magnetohydrodynamics. Awais et al. [27] discussed flow of rate type liquid subject to stretchable surface. Kazemi et al. [28] analyzed convective heat transport of a quiescent liquid by a nonlinear stretchable sheet. Ijaz et al. [29] explored mixed convective nanomaterial flow of viscous liquid with entropy concept. Maria et al. [30] studied radiative flow of magneto nanomaterial subject to exponentially stretchable surface.

The study of chemical reaction in flow by a stretching sheet is significant in biochemical engineering, plastic extrusion and metallurgy. Supplies of moistures and temperature over an agricultural fairness and energy transfer in a drizzly cooling tower are few examples. Krishnamurthy et al. [31], Chamka and Aly [32], Aurangzaib et al. [33] and Pal and Mandal [34,35] studied the chemical reaction effects in flows with different concepts. Rashidi et al. [36] developed Lie group solution for nanofluid flow by a chemically reacted horizontal plate in permeable medium. Abbas et al. [37] studied an upper-convected magneto Maxwell fluid flow through porous medium. In addition mixed convection flow problems have great attention for industrial processes. Muhaimin et al. [38] studied the mixed convection two-dimensional by a flat surface. Mixed convection flows for effects of radiation and MHD are considered in the studies [39,40].

Up to yet there exists no study dealing with chemical reaction in mixed convection three-dimensional flow of Williamson nanofluid. Objective of present article is to study MHD nonlinear mixed convection boundary layer flow of Williamson fluid in three dimensions. Chemical reaction is considered. The relevant problems are solved by converting PDEs into ODEs [41–52]. OHAM [53–55] is utilized for the local similar solutions. The results for various parameters are discussed through graphs and tables. Numerical values for skin friction coefficients and local Nusselt and local Sherwood numbers are computed and addressed.

2. Formulation

Consider steady three-dimensional mixed convection Williamson fluid flow caused by a non-linearly bidirectional stretching sheet. The sheet is aligned with the xy-plane ($z=0$) and the flow occupies the domain $z>0$. Non-uniform applied magnetic field is in z-direction. Sheet temperature and concentration satisfy $T_s > T_\infty$ and $C_s > C_\infty$. Let $u = u_s = a(x+y)^n$ and $v = v_s = b(x+y)^n$ be the nonlinear stretching velocities. Temperature and concentration in present flow are characterized by $T_s(x,y) = T_\infty + T_0(x+y)^n$ and $C_s(x,y) = C_\infty + C_0(x+y)^n$ respectively. Non-linear mixed convection effects are considered. Assume non-uniform magnetic field in the form $B(x,y) = B_0(x+y)^{\frac{n-1}{2}}$. The geometrical sketch of problem is displayed in Fig. 1.

Under standard boundary layer assumptions, the equations for present flow are [11,12,14]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\left.\begin{array}{l} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \nu\frac{\partial^2 u}{\partial z^2} + \sqrt{2}\nu\Gamma\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2} + \frac{1}{\sqrt{2}}\nu\Gamma\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial z^2} - \frac{\sigma}{\rho}B^2(x,y)u \\ +g\{\alpha_1(T-T_\infty) + \alpha_2(T-T_\infty)^2\} + g\{\alpha_3(C-C_\infty) + \alpha_4(C-C_\infty)^2\} \end{array}\right], \tag{2}$$

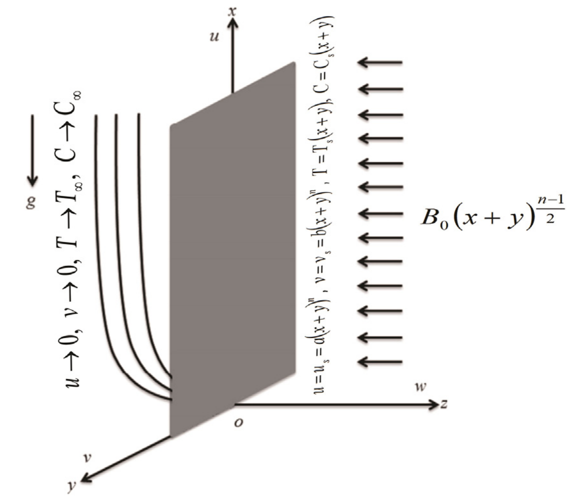

Fig. 1. Geometry of problem.

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \nu\frac{\partial^2 v}{\partial z^2} + \sqrt{2}\nu\Gamma\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial z^2} + \frac{1}{\sqrt{2}}\nu\Gamma\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2} - \frac{\sigma}{\rho}B^2(x,y)v, \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{k}{(\rho C)_f}\frac{\partial^2 T}{\partial z^2} + \tau\left(D_B\frac{\partial C}{\partial z}\frac{\partial T}{\partial z} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial z}\right)^2\right), \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D_B\frac{\partial^2 C}{\partial z^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial z^2} - k_c(C-C_\infty), \tag{5}$$

$$\left.\begin{array}{l} u = u_s(x,y) = a(x+y)^n, \quad v = v_s(x,y) = b(x+y)^n, \quad w = 0, \\ T = T_s = T_\infty + T_0(x+y)^n, \quad C = C_s = C_\infty + C_0(x+y)^n \text{ for } z = 0, \\ u \to 0, \quad v \to 0, \quad T \to T_\infty, C \to C_\infty \quad \text{for } z \to \infty. \end{array}\right\} \tag{6}$$

where u, v and w represent velocity components along $x-, y-$ and $z-$directions respectively, ν the kinematic viscosity of liquid, Γ the material parameter for Williamson fluid, α_1, α_2 the linear and non-linear thermal expansion coefficients, α_3 , α_4 the linear and non-linear concentration expansion coefficients, g the magnitude of gravitational acceleration, σ the electrical conductivity, ρ the density of liquid, k the thermal conductivity, $\tau\left(=\frac{(\rho C)_p}{(\rho C)_f}\right)$ the ratio of liquid to nanoparticles effective heat capacity, D_B and D_T the Brownian and thermophoretic diffusion coefficients, k_c the chemical reaction, a and b the stretching rates, n the power index, T_0, C_0 the dimensional constants and T_s, T_∞ and C_s, C_∞ the temperature and concentration at sheet and far away from sheet respectively.

Employing transformations:

$$\left.\begin{array}{l} u = a(x+y)^n f'(\eta), \quad v = a(x+y)^n g'(\eta), \\ w = -\sqrt{a\nu}(x+y)^{\frac{n-1}{2}}\left(\frac{n+1}{2}(f+g) + \frac{n-1}{2}\eta(f'+g')\right), \\ \eta = \sqrt{\frac{a}{\nu}}z(x+y)^{\frac{n-1}{2}}, \quad \theta(\eta) = \frac{T-T_\infty}{T_s-T_\infty}, \quad \phi(\eta) = \frac{C-C_\infty}{C_s-C_\infty} \end{array}\right], \tag{7}$$

the incompressibility condition is satisfied in a trivial manner and Eqs. (2)–(6) are reduced to

$$\left.\begin{array}{l} f''' + Wef''f''' + \frac{n+1}{2}(f+g)f'' - n(f')^2 - nf'g' + \frac{We}{2}g''g''' \\ -Mf' + \Lambda_T(1+\beta_T\theta)\theta + \Lambda_T N^*(1+\beta_C\phi)\phi = 0 \end{array}\right\}, \tag{8}$$

$$g''' + Weg''g''' + \frac{n+1}{2}(f+g)g'' - n(g')^2 - nf'g' + \frac{We}{2}f''f''' - Mg' = 0, \tag{9}$$

$$\frac{1}{Pr}\theta'' + N_b\theta'\phi' + N_t(\theta')^2 + \frac{n+1}{2}(f+g)\theta' = 0, \tag{10}$$