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Lattice Boltzmann simulation of melting in a cubical cavity with a local heat-flux source

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ABSTRACT

A three-dimensional numerical study is conducted to investigate the effect of heat-flux source location on the melting of phase change material (PCM) in a cubical cavity. An enthalpy-based incompressible thermal lattice Boltzmann model (iTLBM) has been established and validated by two benchmark problems. In order to reveal the effect of heat-flux source location in melting rate in three-dimensional space, both horizontal and vertical locations are considered. The numerical results show that the size of local heater and its heat flux play an essential role in melting rate. Furthermore, the variation of the location of the heat-flux source from both sides to the middle region of the cavity leads to the increase of the total heat flux on the interface, which also results in the increase of melting rate. Finally, we also find that the melting rate increases as the location of the heat-flux source is changed from top to the bottom region of the cavity, which is attributed to the enhancement of convection heat transfer.

1. Introduction

The investigation of heat transfer with solid–liquid phase change has drawn significant attention in applied engineering fields, such as latent heat thermal energy storage [1,2], building walls [3], waste heat recovery [4], battery thermal management [5], and so on. During the past several decades, an extensive literature has been yielded in theoretical, experimental and numerical studies on phase change problem [6,7]. Due to the moving solid–liquid interface and the complexity of heat-momentum-mass mechanisms, it is a high-cost and challenging task to use either experimental or theoretical approaches to investigate such a complicated problem. Consequently, many numerical studies have been performed to study the solid–liquid phase change problem [8,9].

Although there are conduction and natural convection during solid–liquid phase change process, the early works mainly focus on one-dimensional conduction in pure substances. The first study of phase change was performed by Lamé and Clapeyron in 1831 [10], while this type of problems is usually referred to as Stefan problem after Stefan's work [11]. In 1943, London and Seban [12] studied the formation of ice in different geometries (cylinder, sphere, and flat plate), and Meyer [13] performed a comparison between the explicit and implicit methods used in the problem of phase change conduction. Whereafter, Achard et al. [14] carried out an experimental study on a test bench by using an immersed tubular heat exchanger in the PCM and found that there are some significant discrepancies between the experimental results and the theory without including convection effect. Therefore, it is necessary to consider natural convection in the solid–liquid phase change problems. Sparrow et al. [15] first considered the convection heat transfer mechanism in phase change process. Then, to show the influence of natural convection on the melting process, Bareiss [16] performed the melting experiments in a cylindrical vertical test cell with *n*-hexadecane, *n*-octadecane, p-xylene and cyclohexane as test-materials. Recently, Luo [17] conducted a lattice Boltzmann study on the convection–diffusion phenomena associated with solid–liquid phase change process, and also performed a comprehensive analysis on the influences of number and arrangement of tubes on the melting dynamics of shell and tube models.

Generally, the phase change materials have low thermal conductivities which result in low rates of heat storage and retrieval during phase change process [2]. To overcome this problem, the approach through designing suitable heat sources has been considered in many works. Recently, there is a considerable amount of literature on the study of natural convection melting in a square cavity with local heat sources. Zhang [18] was the first to investigate experimentally the melting of *n*-octadecane in a cubical cavity

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with three discrete heat sources flush mounted to the left wall, their results show that natural convection has a significant effect on the shape of solid–liquid interface, and the effect becomes more pronounced with increasing Stefan number. In another experimental study carried out by Zhou [19], the melting process of *n*-octadecane in a rectangular cavity with three discrete protruding heat sources on its bottom surface was investigated. They analyzed the effects of the Stefan number, the sub-cooling and aspect ratio on the melting process. Bruno [20] conducted a numerical study for natural convection dominated melting inside discretely heated rectangular cavity. After that, to explore the capability of PCM for cooling electronic or heat storage applications, Faraji [21] also studied numerically the melting of n-octadecane in a rectangular enclosure heated with three protruding heat source. It is worth noting that the numerical works on PCM melting processes are usually limited to the 2D cases. Bondareva et al. [22,23] studied the natural convective melting in a two-dimensional (2D) and three-dimensional (3D) enclosure containing pure gallium under the effects of inclined uniform magnetic fields and local heater. More recently, Bondareva et al. [24] also analyzed numerically the three-dimensional natural convection melting in a cubical cavity with a local heat source. In their work, the effects of Rayleigh number, Prandtl number and Stefan number on temperature field, velocity field, as well as the average Nusselt number at the heat source surface, have been investigated. Actually, the location of heat source plays an important role in the natural convection heat transfer [25–27]. Chu [25] was the first to study this problem numerically, and found that Nusselt number was proportional to Rayleigh number for any locations of the heat source. Ahmed [26] carried out a numerical study to examine the influence of discrete heat sources on the rate of heat transfer and developed an extrapolation correlation of Nusselt number over a very high range of Rayleigh number. Recently, Darzi [27] investigated the effect of heat source location on the melting of nano-enhanced phase change materials inside a cavity. In their conclusion, when the position of the heat source changes from the top to the bottom of the cavity, the melting rate increases 75 percent. Nevertheless, the numerical study performed by Darzi is also limited to the 2D cases, and the author just considered the effect of vertical location. Moreover, the heat-flux source, which is widely used in electronic devices, should be analyzed comprehensively [28].

In the present work, the effect of heat-flux source location on melting process in a cubical cavity is systematically investigated with a three-dimensional enthalpy-based incompressible thermal lattice Boltzmann model (iTLBM). The capability and accuracy of the iTLBM are tested by some benchmark problems.

The rest of the paper is organized as follows. In Section 2, a lattice Boltzmann model for the phase change problem is introduced. In Section 3, numerical experiments are performed to test the capacity of the present model. In Section 4, the effect of heat-flux source location on melting process has been investigated. Some conclusions are presented in Section 5.

2. Mathematical modeling

2.1. Governing equations

Based on the assumption of the Newtonian, laminar and incompressible fluid flow, the governing equations in describing the phase change problem can be written as [29]

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathbf{uu} = -\nabla p + \nabla \cdot (\nu \nabla \mathbf{u}) + \mathbf{F}, \tag{2}$$

$$\frac{\partial (C_p T)}{\partial t} + \nabla \cdot (C_p T \mathbf{u}) = \frac{1}{\rho_0} \nabla \cdot (\kappa \nabla T) + q, \tag{3}$$

where $\rho_0, \mathbf{u}, p, \nu, C_p, T, \kappa$ are the constant density, velocity, pressure, kinetic viscosity, specific heat, temperature and thermal conductivity. $\mathbf{F}$ is the external force, under the Boussinesq assumption, it can be defined as [30]

$$\mathbf{F} = \mathbf{g}\beta(T - T_{ref}), \tag{4}$$

where $\mathbf{g}$ is the gravitational vector, β is the thermal expansion coefficient, and T_{ref} is the reference temperature. The heat source term q shown in Eq. (3) can be obtained from the latent heat release or absorption,

$$q = -\left[\frac{\partial \Delta H}{\partial t} + \nabla \cdot (\mathbf{u}\Delta H)\right], \tag{5}$$

where ΔH is the variation of latent enthalpy undergoing phase change,

$$\Delta H = f_l L, \tag{6}$$

where f_l and L are the liquid fraction and latent heat of phase change material. The second term of the right-hand side of Eq. (5) can be neglected for the pure phase change material, such that the energy equation becomes

$$\frac{\partial (C_p T)}{\partial t} + \nabla \cdot (C_p T \mathbf{u}) = \frac{1}{\rho_0} \nabla \cdot (\kappa \nabla T) - \frac{\partial f_l L}{\partial t}. \tag{7}$$

2.2. Lattice Boltzmann model for phase change

As a promising numerical method, the lattice Boltzmann method (LBM) [31], which is originated from the lattice gas automata (LGA) method [32], has been widely employed to solve phase change problem for its distinctive advantages as simple calculation procedure, easy implementation of boundary conditions and parallelism [33–38]. In this section, a three-dimensional enthalpy-based incompressible thermal lattice Boltzmann model (iTLBM) is presented.

2.2.1. Incompressible Lattice Boltzmann model for velocity field

Based on the double-distribution lattice Boltzmann model proposed by Shi [39,40], the velocity and thermal fields are described by separate particle distribution functions. In the lattice Boltzmann model for the incompressible Navier–Stokes Eqs. (1) and (2), the evolution equation of the distribution function is written as

$$f_i(\mathbf{x} + \mathbf{c}_i \Delta t, t + \Delta t) = f_i(\mathbf{x}, t) - \frac{1}{\tau_f}[f_i(\mathbf{x}, t) - f_i^{eq}(\mathbf{x}, t)] + \Delta t F_i, \tag{8}$$

where $\mathbf{c}_i$ is the discrete velocity in direction i, τ_f characterizes the relaxation time of the distribution function f_i towards the local equilibrium distribution function f_i^{eq}, which is defined as [41]

$$f_i^{eq}(\mathbf{x}, t) = \lambda_i p + \omega_i \left[\frac{\mathbf{c}_i \cdot \mathbf{u}}{c_s^2} + \frac{\mathbf{uu} : (\mathbf{c}_i \mathbf{c}_i - c_s^2 \mathbf{I})}{2c_s^4}\right], \tag{9}$$

where ω_i is the weight coefficient, c_s is the sound speed, and $\lambda_0 = (\omega_0 - 1)/c_s^2, \lambda_i = \omega_i / c_s^2 (i \neq 0)$.

To study three-dimensional melting problem, the three-dimensional lattice model with nineteen discrete velocity (D3Q19) [42] is employed, and the discrete velocity $\mathbf{c}_i$ is determined as

$$\mathbf{c}_i = c \begin{bmatrix} 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \end{bmatrix}, \tag{10}$$