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Instability mechanisms for thermocapillary flow in an annular pool heated from inner wall

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ABSTRACT

The linear stability analysis of thermocapillary flow in an annular pool heated from inner wall was performed by using spectral element method. The physical instability mechanisms for different Prandtl numbers (Pr) ranging from 0.001 to 1.4 were studied by energy analysis under critical mode. We observed three instability types: (i) the Hopf (oscillatory) bifurcation with wavenumber $k = 18$ for $Pr < 0.02$, and the corresponding neutral mode is amplified due to the basic shear flow; (ii) the stationary instability with $k = 3$ or $k = 2$ for $0.02 < Pr < 0.6$; (iii) the Hopf bifurcation again with $k = 3$ for $0.6 < Pr < 1.4$. In both Prandtl number ranges of $0.02 < Pr < 0.6$ and $0.6 < Pr < 1.4$, thermocapillary force induced by the disturbance temperature field drives primarily the flow instability.

1. Introduction

Thermocapillary flow, driven by the unbalanced surface tension arising from temperature difference, is of great significance in many industrial applications, for example in crystal growth [1]. When the temperature gradient exceeds a threshold value, the two-dimensional (2D) thermocapillary flow becomes unstable, and the crystal quality would be severely affected by such flow instabilities. Therefore, the instability for thermocapillary flow has been widely studied in the past few decades to understand the underlying mechanisms and avoid such flow instabilities.

The cylindrical or annular pool model was widely used to investigate thermocapillary flow. The experimental study of thermocapillary flow in a cylindrical pool was firstly performed by Kamotani et al. [2], and they observed that the oscillatory thermocapillary flow appears when the temperature difference exceeds a certain value. By conducting thermocapillary flow experiments in a cylindrical pool, Ezersky et al. [3] observed a bifurcation from the steady regime to traveling (hydrothermal) waves and then measured the features of the hydrothermal waves. Subsequently, oscillatory thermocapillary flow experiments in cylindrical pool were carried out by Kamotani et al. [4–6] under microgravity, and the temperature distribution and free surface deformation were discussed therein. Besides, Garnier et al. [7] observed the spiral-like hydrothermal waves in an extended annular pool of silicone oil ($Pr = 10$). In the experimental study conducted by Schwabe et al. [8], the thermocapillary flow in the open annular pool with different aspect ratios were investigated, and the critical temperature difference for the onset of temperature oscillation was measured. Later, Shen et al. [9] conducted experiments to investigate the effects of crystal and crucible rotations on the thermocapillary flow in a Czochralski (Cz) configuration with 0.65cSt silicone oil ($Pr = 6.7$). They indicated that the thermocapillary flow with crystal-crucible co-rotation is more stable compared to that of counter-rotation. Moreover, two types of oscillatory waves were observed with the increase of crystal rotation rate. Recently, Kang et al. [10] performed a series of space experiments regarding to thermocapillary flow in an annular pool on SJ-10 satellite, the volume effect for thermocapillary flow, which was investigated early in liquid bridge [11], was investigated under microgravity.

On the other hand, lots of numerical simulation studies have also been devoted to the thermocapillary flows. Sim and Zebib [12] simulated the three-dimensional (3D) thermocapillary flow in an open annular container with unit aspect ratio, and they invesitgated the influence of surface heat loss or rotation on flow instability and convection pattern. Subsequently, Sim et al. [13] numerically investigated the thermocapillary flow in an open cylindrical annuli by adopting the same parameters as in [8] (Schwabe's microgravity experiments), and good agreements were reached. Shi et al. [14] also studied the thermocapillary flow with

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the same geometrical model in [8], and they reported that the gravity stabilized the thermocapillary flow. Later on, Li et al. [15] simulated numerically the thermocapillary flow of silicon melt ($Pr = 0.011$) in a slowly rotating annular pool, and two flow transitions were observed with the increase of the radial temperature difference along the free surface. Recently, the thermocapillary-buoyancy flow of a binary mixture in a shallow annular pool was simulated numerically [16,17], and results indicated that the solute concentration gradient depends on the temperature gradient, and a solute concentration fluctuation appears, accompanying with 3D oscillatory flow. In addition, the flow patterns are influenced by the local interaction between the solutal and thermal capillary effects. The thermocapillary flow in a shallow annular pool with considering surface heat dissipation was performed by Zhang et al. [18], and they reported that with the increase of surface heat dissipation, the thermocapillary flow intensity increases at first and then decreased slightly.

In comparison with the experiments and the direct numerical simulations, the linear stability analysis (LSA) is a more efficient way to capture the critical parameters. Smith and Davis [19,20] applied the LSA on thermocapillary flow in a thin and infinitely extended fluid layer, and they observed two types of thermal-convective instabilities. In a cylindrical liquid bridge for different Prandtl numbers, Wanschura et al. [21] revealed two instability mechanisms for thermocapillary flow: a pure hydrodynamic steady mode for $Pr < 0.1$ melt and an oscillatory hydrothermal wave for $Pr > 0.5$ melt. Subsequently, Shi et al. [22] reported the destabilizing effect of the slow rotation on the thermocapillary flow in a shallow annular pool with silicone oil ($Pr = 6.7$). By using the LSA based on the spectral element method, Yin et al. [23] investigated, recently, thermocapillary flow in a slowly rotating annular pool heated from the outer wall with silicon melt ($Pr = 0.011$). The stability diagram, representing the variation of the critical Marangoni number Ma_c versus the rotation rate ω, exhibits that there is only one Hopf bifurcation point for relative small and large rotation rate, and the energy analysis indicates the corresponding instability is caused by the shear energy, which are provided by the thermocapillary force and pool rotation, respectively. Particularly, the competition between thermocapillary force and pool rotation results in three turning points in the stability diagram for the moderate rotation rate with the increase of Marangoni number. In adddition, the flow instability occurs initially in the zone near the cold wall due to the extreme shear stress. Li et al. [24] performed the LSA on the thermocapillary flow of silicone oil ($Pr = 6.7$) in the rotating annular pools with different aspect ratios. Four types of oscillatory instabilities were reported and the energy analysis revealed that all the instabilities are primarily caused by the hydrothermal wave instability mechanism.

Despite of extensive efforts reviewed above, the physical instability mechanism for the thermocapillary flow in the annular pool has not yet been adequately understood. In this paper, the instability of thermocapillary flow in an annular pool heated from inner wall is investigated by the LSA. In order to capture the critical parameters accurately, the Legendre spectral element method with high degree of accuracy is applied to simulate the 2D axisymmetric steady basic state. The instability mechanisms are then explored by the energy analysis.

2. Problem description and numerical techniques

2.1. Physical and mathematical models

An annular pool is characterized by its depth $d = 12$ mm, inner radius $r_i = 8$ mm and outer radius $r_o = 40$ mm (Fig. 1), and thus the aspect ratio is defined as $\varepsilon = d/(r_o - r_i)$. Constant temperatures are prescribed at the inner (Φ_h) and the outer (Φ_c) cylindrical walls with $\Phi_h > \Phi_c$. The annular pool is filled with melt and the top free surface is assumed to be flat, adiabatic and non-deformable. Herein, the thermocapillary flow is driven by the unbalanced surface tension, and the surface tension is assumed as a linear function of temperature. Besides, the melt is taken as an incompressible Newtonian fluid.

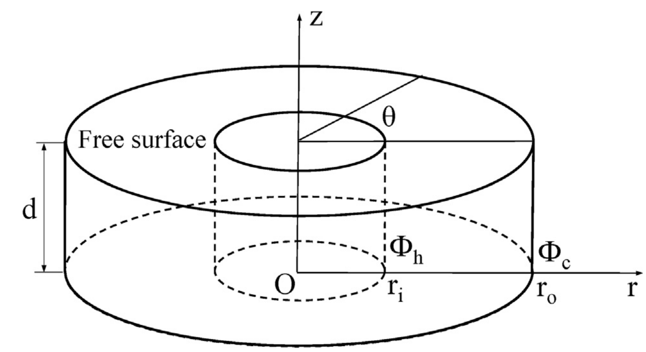

Fig. 1. The geometrical model.

With the above assumptions, the dimensionless governing equations are:

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \nabla^2 \boldsymbol{u} \tag{2}$$

$$\frac{\partial T}{\partial t} + (\boldsymbol{u} \cdot \nabla)T = \frac{1}{Pr}\nabla^2 T \tag{3}$$

where the dimensionless variables, namely the velocity $\boldsymbol{u}$, time t, pressure p and temperature T, are scaled by $\nu/(r_o - r_i)$, $(r_o - r_i)^2/\nu$, $\rho\nu^2/(r_o - r_i)^2$, $(\Phi - \Phi_c)/(\Phi_h - \Phi_c)$, respectively. ρ denotes the fluid density. The Prandtl number is defined by the thermal diffusivity κ and the kinematic viscosity ν as $Pr = \nu/\kappa$.

Besides, the adopted boundary conditions are:

the free surface ($z = \varepsilon = 0.375$)

$$\frac{\partial u}{\partial z} = -\frac{Ma}{Pr}\frac{\partial T}{\partial r}, \quad w = 0, \quad \frac{\partial T}{\partial z} = 0 \tag{4}$$

the bottom ($z = 0$)

$$u = 0, \quad w = 0, \quad \frac{\partial T}{\partial z} = 0 \tag{5}$$

the inner wall ($r = 0.25$)

$$u = 0, \quad w = 0, \quad T = 1 \tag{6}$$

the outer wall ($r = 1.25$)

$$u = 0, \quad w = 0, \quad T = 0 \tag{7}$$

Then, the Marangoni number Ma is defined in terms of the temperature difference as $Ma = \gamma_T(\Phi_h - \Phi_c)(r_o - r_i)/(\rho\nu\kappa)$, where $\gamma_T = -\partial\sigma/\partial\Phi$ is the surface tension coefficient.

2.2. Linear stability analysis

The time splitting method [25,26] based on the Legendre SEM is applied here to solve Eqs. (1)–(7), and the basic state solutions for the assumed 2D axisymmetric steady flow are obtained. Then, the LSA is performed to investigate the onset of flow instability by expressing the solution of the 3D problem as:

$$(\boldsymbol{u}, p, T) = (\boldsymbol{u}_0, p_0, T_0) + (\hat{\boldsymbol{u}}, \hat{p}, \hat{T}) \tag{8}$$