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Numerical inverse method for calculating heat flux in temperature-sensitive-coating measurement on a finite base

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ABSTRACT

The one-dimensional unsteady heat transfer of a polymer layer on a finite base is studied. A numerical inverse method is developed to calculate heat flux from surface-temperature images obtained using temperature-sensitive coating particularly in high-enthalpy wind tunnels. It is found that the effect of the finite thickness of the base is significant as time increases such that the semi-infinite base assumption made in many methods is no longer accurate in heat-flux calculation. The method developed in this work can take into account the effects of the finite thickness of a base and the temperature-dependencies of the thermal properties of the materials. This method has been validated through simulations and processing the experimental temperature-sensitive-paint (TSP) images.

1. Introduction

Global heat-flux measurements in thermoaerodynamics experiments are critical to better understand complex high-speed flow phenomena such as shock-boundary-layer interaction, near-surface stationary vortices, separation, transition and turbulence. Global optical techniques for surface temperature measurements include temperature-sensitive paint (TSP) [1–4], thermographic phosphors (TP) [5–7], thermochromic liquid crystals (TLC) [8–10], and infrared (IR) thermography [11–13]. These techniques have been used to obtain high-resolution surface heat-flux fields in hypersonic wind tunnels. When utilizing TSP, TP and TLC in thermoaerodynamics tests, a thin sensor layer plus a base coating with the relatively low thermal conductivity is usually applied on the surface of a metallic body. These techniques are generally referred to as temperature-sensitive coatings. For IR thermography, a coating with high emissivity is usually applied to the surface to enhance the thermal radiation and signatures associated with flows. From a perspective of inverse heat transfer, IR thermography is also considered as a temperature-sensitive coating technique.

The physical problem in thermoaerodynamics experiments is to determine the surface heat-flux fields from a time sequence of surface-temperature fields measured by using temperature-sensitive coatings. Mathematically, for a given time history of surface temperature of a base (an air vehicle model) with two different layers, it is necessary to determine the corresponding originally unknown surface heat flux by solving the inverse time-dependent heat conduction problem. This problem belongs to a class of boundary inverse heat conduction problems in which an unknown physical quantity at a boundary of a body (e.g. surface heat flux) is inferred from measured temperatures by using sensors at certain locations within the body. The mathematical formulations and various solution methods have been extensively discussed in several books [14–18]. The methodological sketch of the solution to the problem is straightforward. The well-posed positive (forward or direct) heat conduction problem with an initially guessed surface heat-flux function as a boundary condition plus the initial and other boundary conditions is solved to obtain the estimated surface temperature. The norm of the difference between the measured and estimated surface-temperature histories is evaluated as an objective function for optimization. An effective optimization method is applied to obtain the optimal surface heat-flux function by minimizing the objective function. Since the inverse heat transfer problem is ill-posed, to obtain a stabilized solution, the norm is usually modified by adding Tikhonov's regularization term. Several optimization methods have been adopted in different applications, including the Levenberg-Marqaurdt method (damped least-squares method) [18,22], conjugate gradient method [18,19–21], least-squares method with the singular value decomposition [23–24], Tikhonov's regularization method [14], Alifanov's iterative regularization method [15], and Beck's sequential function specification method

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Nomenclature

a	thermal diffusivity
c	specific heat
h_c	convective heat transfer coefficient
k	thermal conductivity
L	thickness
q_s	surface heat flux
Re	real part of complex function
t	time
T	temperature
T_{in}	initial temperature
y	coordinate in normal direction to surface

Greek symbols

θ	temperature change from initial temperature
ρ	density

Subscripts

b	base
p	polymer
ps	polymer surface

[16]. The numerical schemes for the solution of the positive problem include finite difference method, finite volume method, finite element method, boundary element method, differential quadrature method, and spectral method [25].

In principle, the above inverse heat transfer methods developed for various engineering applications can be adapted for this problem associated with temperature-sensitive-coating measurements in aerothermodynamic testing. It is somewhat surprising that researchers in this specialized field tend to use some simple analytical and semi-analytical methods for the one-dimensional (1D) linear inverse heat conduction problem on a semi-infinite homogeneous base. In particular, the Cook-Felderman method has been widely used as a 1D method for a presumed semi-infinite base [26], and other methods along this line have been further developed [27–29]. This situation is partially due to the fact that it is nontrivial to apply the existing inverse heat conduction methods to high-resolution image-based temperature measurements of complex bodies (air and space vehicles). Most existing methods are usually developed and tested for geometrically simple homogeneous bodies (such as a semi-infinite base and a symmetrical body). The 3D inverse heat conduction problem on a complex curvilinear surface with million mesh points (pixels in the image plane) demands very efficient algorithms and powerful computational capability. Furthermore, aerothermodynamics experiments in production wind tunnels require fast image and data processing, which limits the application of computationally-intensive methods.

Here a unique issue in temperature-sensitive coating measurements is that the polymer coating as an optical sensor itself would alter the surface-temperature history, and therefore the effect of the coating on determining the surface heat flux should be accounted for since the base cannot be considered a semi-infinite homogeneous one. Therefore, the Cook-Felderman method and other similar methods are not accurate for these applications. A question is whether the analytical method can be extended. The exact analytical inverse conduction solution for a polymer layer on a semi-infinite base with the constant thermal properties was obtained by using the Laplace transform, in which the effect of the coating is explicitly incorporated [30]. The corresponding numerical inverse method was developed for the problem with the temperature-dependent thermal properties of the polymer and base materials [31]. Further, to correct for the lateral heat conduction effect, the deconvolution method was developed based on the solution of an integral equation of convolution type with a Gaussian kernel [32]. These methods were used to determine the heat-flux fields from TSP measurements for a circular cone in a Mach-6 quiet tunnel [33]. Recently, an exact analytical inverse heat conduction solution for a polymer layer on a finite base with the constant thermal properties was obtained by using the Laplace transform to calculate the surface heat-flux fields from a time sequence of measured surface-temperature fields [34]. Note that the analytical inverse solution is at least marginally stable, that is, a finite error of the measured surface temperature leads to a bounded heat-flux perturbation. Indeed, since the surface temperature is measured, the determination of the surface heat flux is not as ill-posed as the problem with the measured temperature within a body.

As a natural extension of the previous development along this direction, the work will focus on a numerical inverse heat conduction solution for a polymer layer on a finite base with temperature-dependent thermal properties since no analytical solution exists in this case. In general, the thermal properties of the polymer and base materials are sensitive to temperature [35]. The heat-flux calculation with the assumed constant thermal properties would have an error particularly when the surface temperature change is large depending on a specific air vehicle model and the test conditions. Fig. 1 shows a polymer layer with the thickness of L_p on a base with the thickness L_b. In TSP measurements, the TSP layer and the insulating base layer are both polymers with similar thermal properties, and thus are treated as a single layer in the inverse heat conduction analysis [1–3]. The first step is to solve the positive heat transfer problem for the geometry shown in Fig. 1, which is required not only for simulations but also for the numerical inverse heat-flux estimation algorithm. The inverse heat transfer method starts from the initial determination of the heat flux by using a simple and fast approximation calculation. Then, an iterative algorithm refines estimation through an optimization scheme. The inverse heat transfer algorithm is examined through simulations. Given a simulated heat transfer history and the temperature-dependent thermal properties, the change in the surface temperature of the polymer layer on a finite base (either Al or Nylon6) is obtained accurately by numerically solving the heat conduction equation as a positive problem. Then, from the simulated change in the surface temperature, the inverse heat transfer algorithm is used to recover the heat flux for a direct comparison with the known heat-flux history and the result given by using the analytical inverse method. Applications of the numerical inverse heat conduction method in TSP measurements in hypersonic wind tunnels are also described.

2. Positive heat conduction problem

The mathematical formulations are the same as those in the analytical solution [34]. For convenience of reading, the main equations with the initial and boundary conditions are recapitulated here. As shown in Fig. 1(a), a polymer layer with the thickness of L_p on a base with the thickness of L_b is considered in our