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Near-wall resolution requirement for direct numerical simulation of turbulent flow using multidomain Chebyshev grid

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A B S T R A C T

Direct numerical simulation (DNS) of turbulent flow in a periodic channel is performed to study the effect of the wall-normal spatial resolution near the wall on the calculations of turbulence statistics using multidomain Chebyshev grids. A discontinuous spectral element method (DSEM) is employed to calculate the first- and second-order statistics of the flow near the wall. The effect of the spectral approximation order on the resolution requirement is also studied by considering three approximation orders of $P = 2,\ 5$, and 7. The Reynolds number based on the bulk density, bulk velocity, and channel half-height is $Re = 3266$, which corresponds to a friction Reynolds number of $Re_\tau \approx 204$ based on the wall friction velocity and the channel half-height. It is observed that the near-wall resolution requirement strongly depends on the spectral approximation order. For the same total number of grid points, a higher approximation order provides more accurate results. For approximation orders of $P = 5$ and 7, grids with respectively 11 and 8 points inside $y^+ = 10$ are sufficient to resolve the turbulent statistics near the wall, while a grid with $P = 2$ requires more than 11 points in the same region to achieve the same level of accuracy.

1. Introduction

The accuracy of the predictions of turbulence statistics near the wall in direct numerical simulation (DNS) of turbulent flows depends strongly on the grid resolution near the wall, especially in the wall-normal direction. In DNS of near-wall turbulence, the resolution near the wall should be sufficient to capture the behavior of the boundary layer in all three regions of the inner layer: the viscous (laminar) sublayer, the buffer layer, and the log-law (fully turbulent) region. It has been shown that an under-resolved grid in one or more directions negatively affects the prediction of the mean properties as well as higher-order statistics of the flow [1]. Since the grid spacing criteria should be independent of the Reynolds number, the spatial spacing is usually calculated based on the non-dimensional wall unit, y^+. Previous grids used for DNS of wall-bounded turbulent flows usually satisfy two general conditions: (i) The nearest grid point to the wall is located below $y^+ = 1$, and (ii) there are at least 10 grid points within $y^+ = 10$ (hereinafter, called y^+_{10}). For most grids, these conditions can be satisfied by placing the first point below $y^+ = 1$ and gradually increasing the grid spacing (using a geometric progression for example) as moving away from the wall. However, in numerical schemes that use a Chebyshev distribution of grid points, satisfying the second condition would result in a minuscule grid spacing at the wall (usually, orders of magnitude smaller than $y^+ = 1$) due to the high non-uniformity of point distribution. Some types of spectral and spectral element methods are examples of the schemes that use Chebyshev grids, which have been increasingly used by researchers recently [2,3].

For example, the discontinuous spectral element method (DSEM) [4–6] divides the physical domain of interest into non-overlapping hexahedral subdomains (the so-called elements). The hexahedral elements, which may have edges with arbitrary lengths, are then mapped to a unit cube in the computational space. Inside each element, the spectral approximation is applied on a staggered grid with a Chebyshev distribution of points. The grids consist of two staggered sets of points: Gauss quadrature points (used to calculate the solutions) and Gauss-Lobatto quadrature points (used to calculate fluxes). The Gauss quadrature points in the mapped space are placed in each direction using a symmetric Chebyshev distribution of the form

$$X_{i+\frac{1}{2}} = \frac{1}{2}\left[1 - \cos\left(\frac{i+\frac{1}{2}}{P+1}\,\pi\right)\right], \quad i = 0, \ldots, P, \qquad (1)$$

over the interval of $[0, 1]$, where P is the spectral approximation order [4]. For an approximation order of $P = 9$, such element has 10 Gauss quadrature points with the distribution shown in Fig. 1.

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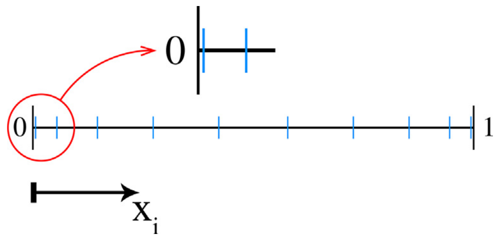

Fig. 1. Distribution of Gauss quadrature points (blue ticks) in a Chebyshev grid of order $P = 9$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Note that the first point is located at $X_{1/2} = 6.16 \times 10^{-3}$. If the near-wall elements are distributed normal to the wall such that the first point off the wall is located at $y^+ = 1$, there would be only 2 points inside y^+_{10}. Even by placing the first point at $y^+ = 0.1$, there would be only 6 points within y^+_{10}. On the other hand, if the elements near the wall are placed such that the second condition is satisfied, i.e., there are 10 Gauss points within y^+_{10}, the first point near the wall would be located at $y^+ = 0.0616$. In this case, the extremely small grid size, which occurs both on the wall side and on the other end of the first element, severely restricts the time step size and makes the simulation computationally expensive if the numerical stability is conditional on the time step size. Therefore, for such schemes, it is challenging to determine the sufficient near-wall resolution, while minimizing the restriction of the time step size. To the best knowledge of the authors, this challenge has not been tackled so far.

The periodic channel flow is a well-studied benchmark for wall-bounded turbulent flows and is used as the test case here. In this paper, a series of DNS of periodic turbulent channel flow are conducted using DSEM to study the requirement of the grid resolution normal to the wall for accurate prediction of turbulent statistics using multidomain Chebyshev grids. The effect of the spectral approximation order on the spatial resolution requirement is also studied by testing three different orders. The flow is simulated using nine grids (three grids with different resolutions for each approximation order), and their mean flow statistics, velocity fluctuations, and Reynolds shear stresses are compared.

1.1. Previous DNS of channel flow

The DNS of turbulent flow in a periodic channel has been intensively studied. Kim et al. [7] conducted DNS of an incompressible, turbulent channel flow using a spectral method. The simulations were performed at a friction Reynolds number of $Re_\tau = u_\tau\delta/\nu = 180$, where u_τ is the wall friction velocity, δ is the channel half-height, and ν is the fluid's kinematic viscosity. Their scheme featured a fully spectral method with Fourier series in the homogeneous directions, i.e., streamwise and spanwise directions, and a Chebychev polynomial expansion in the normal direction. The grid that they used had 12 points inside y^+_{10}, and the nearest grid point to the wall was located at $y^+ = 0.05$. Rai and Moin [8] presented a finite-difference (FD) solution to the incompressible fully developed turbulent channel flow at a friction Reynolds number of $Re_\tau = 180$ and provided a comparison between the results obtained using FD and spectral methods. They used a geometric progression for the distribution of near-wall grid points normal to the wall. Lyons et al. [9] presented results from a DNS of fully developed incompressible turbulent flow in a channel. They also used Fourier series expansions in the periodic directions and Chebyshev polynomial expansions in the normal direction. The friction Reynolds number was $Re_\tau = 130$, and the nearest grid point to the wall was located at $y^+ = 0.18$. Crawford [10] employed a spectral element method and conducted a grid resolution study for an incompressible turbulent flow in a periodic channel at a friction Reynolds number of $Re_\tau \approx 207$. The grid that was concluded to be adequate to resolve the turbulent statistics had its first point near the wall at $y^+ = 0.29$. Moser et al. [11], as the continuation of the work of Kim et al. [7], used the same code later and reported detailed statistical data from DNS of incompressible fully developed turbulent channel flow at three friction Reynolds numbers of $Re_\tau = 180$, 395, and 590. In all three cases, they had 13 or more grid points within y^+_{10}. del Alamo and Jimenez [12] performed DNS of turbulent channel flow at Reynolds numbers of $Re_\tau = 180$ and 550 using the same numerical method as [7]. Their focus was on the size and location of large scales of motion in the channel flow. Morinishi et al. [13] performed DNS of turbulent channel flows using an algorithm based on the B-spline collocation method in the wall-normal direction and the Fourier Galerkin method in the periodic directions. They considered both incompressible and compressible (Mach number of 1.5) cases at a Reynolds number of $Re_f = 3000$ based on the bulk density, bulk velocity, and channel half-height. They used a hyperbolic-tangent function for the distribution of the wall-normal collocation points, and the nearest point to the wall was located at $y^+ = 0.045$ and ~ 0.35 for their incompressible and compressible cases, respectively. Lee and Moser [14] performed DNS of incompressible channel flow at different Reynolds numbers ranging from $Re_\tau = 180$ to 5186. They used a Fourier-Galerkin method in the periodic directions and a B-spline collocation method in the wall-normal direction. The grid they used for the highest Reynolds number had 15 points within y^+_{10}. The first points for the lowest and the highest Reynolds number cases were located at $y^+ = 0.074$ and $y^+ = 0.498$, respectively. A summary of the above-mentioned studies along with more details of their computational domains and grids are included in Table 1.

Grötzbach [15] deduced three criteria for the prediction of grids for DNS of turbulent flow. Their second criterion, concerning the near-wall resolution, states that, for turbulent flows with a Prandtl number below unity, at least three grid points must be placed inside the viscous sublayer ($y^+ < 5$). Later, Moin and Mahesh [16] provided a review of DNS of turbulent flows. They pointed out that spectral schemes require less spatial resolution than other schemes, such that second-order central difference schemes need about twice the resolution (in each direction) to achieve the same level of accuracy as a spectral DNS.

In the following section, the governing equations and the numerical methodology that are used for the present simulations are provided. Next, the problem setup for the periodic channel flow and the grid generation procedures are discussed. Then, the results of the simulations and discussions are presented; first, it is shown that the statistics of interest are not affected by the compressibility, and the methodology is able to reproduce previous DNS results. Then, the mean velocity, temperature, and density profiles, as well as mean flow variables, are compared for different cases. Furthermore, the second-order statistics including the velocity fluctuations and Reynolds shear stresses are presented and discussed for all cases. Next, the cases are compared based on their computational costs. Finally, conclusions are drawn.

2. Governing equations and numerical methodology

2.1. Governing equations

The governing equations for three-dimensional (3D), unsteady, compressible, viscous fluid flow, the so-called full Navier-Stokes equations, consist of the conservation of mass, momentum, and energy. The non-dimensional form of these equations in the conservative form are presented in the Cartesian vector notation as