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Numerical solutions of free convection flow of nanofluids along a radiating sinusoidal wavy surface

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ABSTRACT

In this article, our interest is to investigate the free convection boundary-layer flow of a nanofluid past a sinusoidal semi-infinite vertical surface in the non-absorbing medium. The contribution of surface radiative heat flux is employed in the boundary conditions with the help of Stefan-Boltzmann law. Governing equations are transformed into a non-conserved dimensionless system and integrated numerically by means of an implicit finite difference scheme. The accuracy of numerical method is assessed by a comparison of the present results with the earlier published work and it is found that there exists a favorable agreement between the two. Our problem is subject to the influence of a set of parameters, which appear due to the presence of surface radiation, nanoparticles, and transverse curvature of vertical wall. It is noticed that radiative length parameter λ and surface radiation parameter R contribute in a reduction of local skin friction coefficient and local Nusselt number.

1. Introduction

In recent years, advances in the area of nanotechnology led to the development of nanofluids produced by dispersing the submicron solid particles in liquids. The term "nanofluid" was initially introduced by Choi [1] in order to present the engineered colloidal suspensions of nanoparticles in a base fluid. In recent years nanofluids are used extensively, which warrant their participation in numerous practical applications, for instance, nanofluids are being used as a cooling agent for nuclear system at Massachusetts Institute of Technology (MIT), in transportation industry, nanomaterial processing, micro-electromechanical systems (MEMS), polymer coating industry, microbial fuel cell technology, and medical systems (*i.e.*, cancer therapeutics, delivery of nano-drugs and cryopreservation) (see [2]). The nanoparticles are composed of the materials such as chemically stable metals (gold, copper, etc.), oxides (silica, alumina, copper, titania, etc.), nitrides (phosphorus, boron, etc.) and/or non-metallic materials. Most commonly used base fluids for preparation of nanofluids are conductive liquids, water, organic fluids (*i.e* glycol, ethanol, etc.), polymer solutions, lubricants and bio-fluids. Nanofluids are also expected to transfer heat at a higher rate than ordinary fluids [3,4]. Buongiorno [5] physically explained the abnormal convective heat transfer enhancement observed in nanofluids. A detailed review article was presented by [6] in order to summarize the important published articles on the enhancement of the forced convection heat transfer with nanofluids. There have been published several recent papers [6–13] on the mathematical modeling of convection heat transfer in nanofluids.

Convection problems with irregular surfaces (sinusoidal wavy) find numerous industrial and engineering applications due to their capability of promoting higher heat transfer rate (typical examples includes solar collectors, industrial heat ex-changers, grain storage containers, electrical and nuclear cooling components, chemical catalytic reactors and condensers in refrigerators). The idea of convective boundary layer flow along wavy surfaces was first introduced by Yao [14] for the case of uniform surface temperature. The case of uniform surface heat flux, which is often approximated in practical applications, was investigated by Moulic and Yao [15]. They showed that the Nusselt number varies periodically along the wavy surface. The study of natural convection along an isothermal wavy cone embedded in a fluid-saturated porous medium was numerically investigated by Pop and Na [16]. The effects of wavy surface on natural convection over a vertical frustum of a cone was studied by Pop and Na [17] in which the authors presented detailed results for the local Nusselt number and wall temperature for a selection of parameters consisting of the wavy surface

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Nomenclature

$\bar{a}$	dimensional amplitude of wavy surface
a	dimensionless amplitude of the wavy surface
C	dimensionless concentration of nanoparticles
C_w	dimensionless concentration of nanoparticles at the surface
c_p	specific heat at constant pressure for base fluid
c_{np}	specific heat at constant pressure for nanoparticles
C_f	skin friction coefficient
D_B	Brownian diffusion coefficient
D_T	Thermophoretic diffusion coefficient
g	gravitational acceleration
Gr	Grashof number
L	characteristic length associated with wavy surface
Ln	Nanoparticle Lewis number
$\vec{n}$	unit normal vector to wavy surface
N_A	modified diffusivity ratio parameter
N_B	particle-density increment parameter
Nr	buoyancy ratio parameter
Nu	Nusselt number coefficient
$\bar{p}$	dimensional pressure
p	dimensionless pressure
Pr	Prandtl number
q_w	uniform heat flux
q_{np}	nanoparticle flux
R	dimensionless surface radiation parameter
S	dimensionless concentration of nanoparticles in parabolic coordinates
T	dimensional temperature of fluid
T_w	dimensional temperature at the surface
T_∞	ambient fluid temperature
$\bar{u},\ \bar{v}$	dimensional fluid velocities in $(\bar{x},\ \bar{y})$ direction
$u,\ v$	dimensionless fluid velocities in $(x,\ y)$ direction
$U,\ V$	dimensionless fluid velocities in $(X,\ Y)$ coordinate system
$\bar{x},\ \bar{y}$	dimensional Cartesian system
$x,\ y$	dimensionless coordinate system
$X,\ Y$	dimensionless parabolic coordinate system

Greek letters

α	thermal diffusivity of nanofluid
β	volumetric expansion coefficient of the nanofluid
κ	thermal conductivity of nanofluid
ϵ	emissivity
θ	dimensionless fluid temperature
τ	ratio of heat capacity of nanofluid to the heat capacity of the base fluid
ϕ	dimensional concentration of nanoparticles
ϕ_w	nanoparticle volume fraction at the surface
ϕ_∞	nanoparticle volume fraction in free stream
ρ	density of base fluid
ρ_{np}	density of the nanoparticles
μ	dynamic viscosity
ν	kinematic viscosity
Θ_w	surface temperature parameter
$\bar{\sigma}(\bar{x})$	dimensional surface profile function
$\sigma(x)$	dimensionless surface profile function
σ_x	first derivative of the function σ with respect to x
σ_{xx}	second order derivative σ with respect to x
σ_e	Stefan-Boltzmann constant
λ	radiative length parameter

Subscripts

w	surface condition
∞	ambient condition
np	nanoparticles condition

Superscripts

–	dimensional system

amplitude, half cone angle and Prandtl number. Very recently, the influence of nanofluids over the boundary layer flows along roughened geometries was numerically examined by Siddiqa et al. [18,19].

Heat transport calculation for the equipments having high-temperature surfaces necessitate to consider the simultaneous influence of various mechanisms of heat transfer on characteristics of the process. Of major interest is the interaction of free convection with radiation. This interaction can be established in many ways, for instance, (i) by coupling energy equation with thermal radiation heat flux term, (ii) by introducing a separate governing equation for radiation, and (iii) and also by assigning a heat radiating boundary condition on surface. Some of the initially reported studies [20–23] observed the thermal radiation effects by using Rosseland diffusion approximation. This diffusion approximation is precisely applicable in interior part of the medium and not close to the objects, therefore, the work in [20–23] is accompanied by considering the scattering, emissivity and absorption of radiation in the fluid. Further, [24–26] introduced the Rosseland approximation in the boundary conditions in order to incorporate the radiation effects within the surface/object. Therefore in these papers scattering, emission and absorption processes were ignored and the surface transmits a uniform thermal flux into the participating optically transparent medium. This idea was further used by Siddiqa et al. [27,28] under different physical situations.

From the literature survey it was found that heat transfer problems of nanofluid flows along uniform and/or roughened surfaces have been discussed by various investigators but the influence of surface radiation on nanofluid flow was not reported earlier. Therefore, in this paper we establish the solutions of nanofluid flow along a wavy surface emitting complex radiations in boundary-layer regime. Our work is based on the assumptions that the process of thermophoresis and Brownian motion are responsible for the motion of nanoparticles. Stefan-Boltzmann law for boundary conditions is used to incorporate the effect of radiation. Our goal is to investigate the combined influence of surface radiation and natural convection on a wavy pattern in the water based nanofluid. This physical situation is interpreted mathematically with the help of Navier-Stokes equations coupled with energy equation for the volume fraction nanoparticles moving over the wavy surface. Primitive variable formulation is used to transform the governing equations into a suitable form over which two-point finite difference scheme is applied for obtaining solutions. For validation of our scheme, comparison is made with Moulic and Yao [15] and is found in good agreement. Numerical simulations are performed up to the desired level of accuracy and parametric solutions are presented for skin friction coefficient, rate of heat transfer, streamlines, velocity, temperature and concentration profiles.

2. Problem formulation

We consider viscous, steady, incompressible, free convection flow of a nanofluid along a thermally radiating semi-infinite