International Journal of Heat and Mass Transfer 126 (2018) 908–925

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Unified framework for buoyancy induced radiative-convective flow and heat transfer on hybrid unstructured meshes

Mukul Parmananda, Amaresh Dalal *, Ganesh Natarajan

Department of Mechanical Engineering, Indian Institute of Technology Guwahati, Guwahati 781039, India

ARTICLE INFO

Article history:
Received 1 February 2018
Received in revised form 26 April 2018
Accepted 17 May 2018

Keywords:
Non-Boussinesq convection with thermal radiation
Low-Mach number formulation
Unstructured meshes

ABSTRACT

The development of a non-Boussinesq flow solver for simulating combined radiative-convective heat transfer is presented on arbitrary polygonal meshes using the ideas of low-Mach number asymptotics. A segregated approach for solving the governing equations using a fractional step methodology on finite-volume method is adapted to handle the low-Mach number formulation. Simulations are carried out for two- and three-dimensional problems involving combined convective-radiative heat transfer both in the small and large temperature difference regimes. It is shown through investigations over a range of Gay-Lussac's and Planck numbers that non-Boussinesq effects could become significant due to the sole influence of large temperature difference and radiative heat transfer. Furthermore, the influence of non-Boussinesq effect on overall heat transfer is larger for the three-dimensional simulation of a given radiation-convection heat transfer problem relative to the two-dimensional assumption for the same problem. Interestingly, even at the low-temperature difference with the presence of significant radiation the Boussinesq approximation fails. This study clearly identifies the limits of validity of the Boussinesq approximation and conclusively prove that solutions to radiation-convection heat transfer problems are best acquired using a quasi-incompressible approach as described in this work.

1. Introduction

Radiative heat transfer coupled with natural and mixed convection has many significant applications like electronic cooling, heat transfer, the thermal analysis in nuclear reactors, fire and plume assemblies, thermal insulation systems, open fire and combustion, etc. Accurate design and efficient optimization of these applications require a numerical approach capable of handling coupled heat transfer with the flow in conjunction with radiation. Due to the non-linearity associated with thermal radiation at elevated temperature difference, choice of a relevant numerical model becomes vital. There have been many researchers who have presented various models for accurate simulation of combined natural convection with thermal radiation. The pioneering works in this context [1–3], have presented various benchmark results for combined radiation-convection heat transfer in enclosures.

The numerical methods of radiative-convective heat transfer require the solution of radiative transfer equation (*RTE*) in conjunction with the flow. Use of *FVM*, *DOM*, and *LBM* for radiation with conventional methods for flow have emerged as a popular technique with [1–4] adopting discrete ordinate method (*DOM*) whereas [5–8] using *FVM* for coupled volumetric radiation and natural convection problems in enclosures. Researchers like [9–12] used *LBM* for studying natural and double-diffusive convection with volumetric radiation in enclosures. Despite extensive study on buoyancy-driven convection heat transfer inside an enclosure, the studies on combined natural and mixed convection with radiative heat transfer are limited. Ahmed et al. [13] presented a coupled natural convection with radiative heat transfer in inclined porous cavities, while Ibrahim et al. [14] studied buoyancy driven turbulent convection under the influence of radiation. There are few studies on three-dimensional convection with gas and soot radiation by [15–17] which highlight the three-dimensional flow and heat transfer due to the presence of radiation. Martyushev and Sheremet [18] performed a three-dimensional simulation to study the influence of a heat source located at the base of the cavity on average convection and radiation Nusselt number. Kolsi et al. [19] studied the influence of aspect ratio on natural convection with radiative heat transfer in a three-dimensional enclosure. They demonstrated three-dimensionality of the flow with radiation and quasi-two-dimensional flow without radiation. Lei and Patterson [20] studied the flow and heat transfer in a three-dimensional shallow wedge with solar radiation-induced natural convection by performing a direct numerical analysis.

* Corresponding author.
E-mail addresses: p.mukul@iitg.ernet.in (M. Parmananda), amaresh@iitg.ernet.in (A. Dalal), n.ganesh@iitg.ernet.in (G. Natarajan).

https://doi.org/10.1016/j.ijheatmasstransfer.2018.05.092
0017-9310/

Nomenclature

A_f	area of a given face (m^2)
c_p	specific heat at constant pressure $\left(\frac{J}{kg\,K}\right)$
c_v	specific heat at constant volume $\left(\frac{J}{kg\,K}\right)$
D	downwind cell
D_f^m	directional weight
g	acceleration due to gravity $\left(\frac{m}{s^2}\right)$
Ga	Gay-Lussac number
H	enclosure height (m)
I	intensity of radiation $\left(\frac{W}{m^2\,sr}\right)$
I_b	intensity of radiation for a black body $\left(\frac{W}{m^2\,sr}\right)$
k	thermal conductivity $\left(\frac{W}{m\,K}\right)$
$\hat{n}_f$	unit vector normal to the face
N	neighbor cell of a face
Nu	Nusselt number
Δn	distance between cell P' and N' (m)
p	hydrodynamic pressure $\left(\frac{N}{m^2}\right)$
$\overline{P}$	thermodynamic pressure $\left(\frac{N}{m^2}\right)$
P	owner cell of a face
Pl	Planck number
Pr	Prandtl Number
$\mathbf{q}_r$	heat flux due to radiation $\left(\frac{W}{m^2}\right)$
$\mathbf{r}$	position vector
Ra	Rayleigh Number
R	universal gas constant $\left(\frac{J}{mol\,K}\right)$
$\mathbf{s}$	direction vector
T	non-dimensional temperature
Δt	time step
x, y, z	dimensionless co-ordinates
u, v, w	dimensionless velocity components along x, y and z
V_P	volume of the cell P (m^3)

Greek symbol

α	thermal diffusivity $\left(\frac{m^2}{s}\right)$
β	extinction coefficient (m^{-1})
β_T	volumetric expansion coefficient (K^{-1})
ϵ	Boussinesq parameter
κ_a	absorption coefficient (m^{-1})
ε_w	wall emissivity
ν	kinematic viscosity $\left(\frac{m^2}{s}\right)$
ρ	density $\left(\frac{kg}{m^3}\right)$
σ_b	Stefan-Boltzmann constant $\left(\frac{W}{m^2\,K^4}\right)$
σ_s	scattering coefficient (m^{-1})
τ	optical thickness (βH)
Υ	volume of the enclosure (m^3)
ω	scattering albedo $\left(\frac{\sigma_s}{\beta}\right)$
Ω	solid angle (sr)
Φ	scattering phase function

Subscript

bf	total number of faces in boundaries
f	face
NC	total number of cells
c	cold
h	hot
r	radiative term
w	wall
o	reference state

Superscript

$*$	provisional momentum
m	discrete direction of intensity
$n+1$	present time level
n	previous time level
$n-1$	previous to previous time level

Abbreviations

DOM	discrete ordinate method
FVM	finite volume method
LBM	lattice Boltzmann method
LMN	low-Mach number
LIS	library of iterative solver
RTE	radiative transfer equation

Numerous studies have been performed on natural and mixed convection flows in enclosures with a heat generating cylinder due to its applicability for cooling of equipment used in nuclear engineering. Mezrhab et al. [21] examined buoyancy-induced convection in an enclosure with a heated square cylinder. Later, Mezrhab et al. [22] extended the work for studying the effects of the circular heated cylinder inside a cavity. Xu et al. [23] and Xing et al. [24] studied heated cylinder inside a circular and a cylindrical enclosure of varying shape. Most of the studies on buoyancy-driven convection considered a low-temperature difference and Boussinesq approximation was used to handle the density variation. In spite large volume of work, the use of non-Boussinesq models is limited to pure convective heat transfer problems using low-Mach number formulation like [25–27]. The coupled convection-radiation heat transfer using non-Boussinesq formulation has been scarcely studied by [5,28,29]. In the present work, authors have presented a low-Mach number solver for simulating non-Boussinesq flow and heat transfer following the work of Paolucci [30] and further considering the radiation effects.

While there have been limited studies on convective heat transfer at large temperature difference by [31–34], there have been no significant efforts to understand the validity of Boussinesq approximation in radiative-convective heat transfer. Typically many works have routinely employed the Boussinesq approximation [1–4,9–14] despite the fact that it does not remain valid in many scenarios which are particularly highlighted in the present work. The present study investigates the validity of Boussinesq approximation for two- and three-dimensional buoyancy driven flows under the influence of thermal radiation. The authors have presented different scenarios considering both small and large temperature difference to determine the validity of Boussinesq approximation and its impact on numerical results. Along these lines, it is shown that the presence of significant radiation even at small temperature difference can render the Boussinesq approximation invalid. Furthermore, the non-Boussinesq effects exhibit a strong influence on overall heat transfer in a three-dimensional simulation as compared to a two-dimensional approximation of the same problem. Discrepancies in the results of Boussinesq and non-Boussinesq models are highlighted by considering two test cases, representative of practical scenarios encountered in many applications like the flow of air and smoke in building fires, cooling of electronic and nuclear equipment. The inaccuracy of the incompressible model to simulate flow and heat transfer observed in the various practical scenario involving convective-radiative heat