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Review

Evolutionary design of conducting layers with fins and freedom

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ABSTRACT

Here we show that the morphology of a conducting wall can be 'evolved' by design so that its ability to enhance thermal contact with the ambient is maximized. Free to morph is the entire configuration of the conducting volume of the wall: the wall thickness and the population of fins planted on the wall are free to vary. The evolution of the design is pursued systematically. First, the thickness of the base layer is free to morph, and the population of fins is uniform (identical fins are spread equidistantly). Second, the base thickness and the fin-to-fin spacings are free to morph, while the fins are identical. Third, the base thickness and the fin sizes are free to morph, while the fins are distributed equidistantly. The emerging rules for distributing the wall thickness and the fin size and density are reported. The optimal allocation of the conducting material to the longitudinally conducting wall and the finned structure is also reported.

Contents

1. Morphing with freedom

The enhancement of heat transfer for cooling applications has generated a wide variety of surface configurations, such as extended surfaces (fins) and 'heat spreaders', which are conducting plates that make the wall temperature more uniform in the vicinity of the heat sources that reside on the wall. The literature on such techniques is voluminous: Recent reviews were provided by Yovanovich and Marotta [1] and Aziz [2]. Examples from the most current literature in this domain are Refs. [3–17].

In this paper we consider the techniques of heat spreading and finning as one technique (Fig. 1), and we approach it from the point of view of evolutionary design [18,19]. According to the constructal law, improvements in the overall performance of a flow system follow as a result of endowing the flow system with more freedom to morph. This means two things: (1) the plate (the base) that conducts heat away from the source, along the wall, is free to morph its thickness, and (2) the population of fins that are installed on the base is free to morph as well. The fins are not necessarily identical and spread equidistantly.

The work that is described in this article is conceptual and theoretical. It is presented systematically in three parts, from the simpler toward the more complicated. First, the thickness of the base

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Nomenclature

Bi	Biot number, Eq. (26)
g	gravitational acceleration, ms^{-2}
h_b, h_{b*}, h_{b**}	apparent base heat transfer coefficient, W m^{-2} K^{-1}, Eqs. (1), (27) and (46)
h_f	heat transfer coefficient on the fin surface, Eq. (12), W m^{-2} K^{-1}
H	fin height, m
k	solid thermal conductivity, W m^{-1} K^{-1}
k_a	fluid thermal conductivity, W m^{-1} K^{-1}
L	half-length of base layer, m
m, n, p, r	exponents
N'	number of fins per unit of base length, m^{-1}
q_1	heat current drawn by one fin from the base layer, W, Eq. (13)
q''	heat flux out of the upper surface of the base, W m^{-2}
Q	total heat current, W, Eq. (9)
S	fin spacing, m
t	fin thickness, m
T	base excess temperature above the ambient, K
V_{base}	volume of base layer, m^3
V_{fins}	volume of all the fins, m^3
V_{tot}	solid volume constraint, m^3, Eq. (19)
W	base width, or fin length perpendicular to Fig. 1, m
x	coordinate along the base layer, m, Fig. 1
z	volume fraction occupied by the base layer, Eq. (20)

Greek symbols

α	thermal diffusivity, m^2 s^{-1}
β	coefficient of volumetric thermal expansion, K^{-1}
δ	thickness of base layer, m
$\bar{\delta}$	average base thickness, m
ΔP	pressure difference, N m^{-2}
μ	viscosity, kg s^{-1} m^{-1}
ν	kinematic viscosity, m^2 s^{-1}
ξ	dimensionless coordinate along the base, x/L

Subscript

$()_*$	value at the heat source location ($x = L$)

layer is free to morph, and the population of fins is uniform (identical fins spread equidistantly). Second, the base thickness and the fin-to-fin spacings are free to morph, while the fins are identical (Fig. 2). Third, the base thickness and the fin sizes are free to morph, while the fins are distributed equidistantly (Fig. 3).

2. The base layer

Consider a highly conductive thin layer of solid attached to a concentrated heat source with the purpose of spreading the heat current along the layer, and then sideways by convection (forced or natural) to the ambient. The transfer of heat by convection is facilitated by a population of fins mounted perpendicularly on the conductive layer.

The objective of the following analysis is to determine the main features of the solid architecture—conductive layer and fins—that offer greater access to the flow of heat from the source to the ambient through the solid structure. This global objective is the same as determining the class of architectures for which the temperature difference between heat sink and ambient is minimal when the heat current generated by the source is specified.

The main features of the proposed family of designs are captured by the drawing made in Fig. 1. For now, we separate the conductive layer from its fins, and at the interface between these two

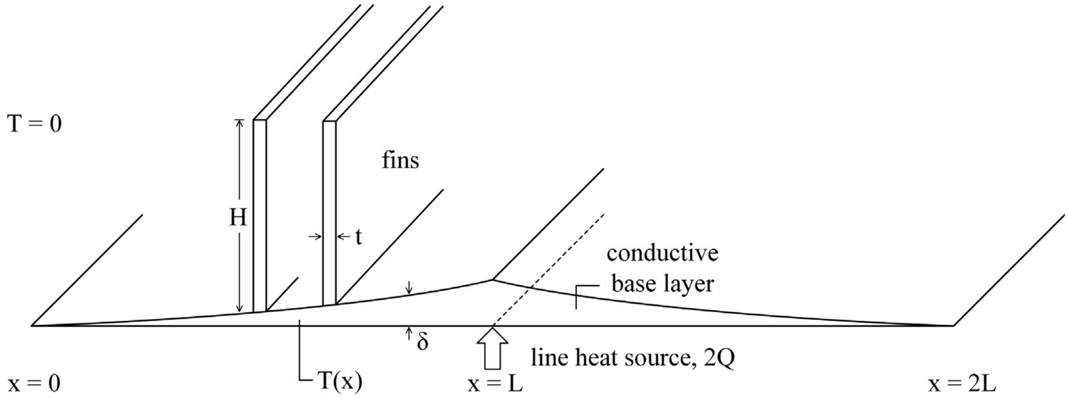

Fig. 1. Conductive base with nonuniform thickness and uniformly spread fins for convection.

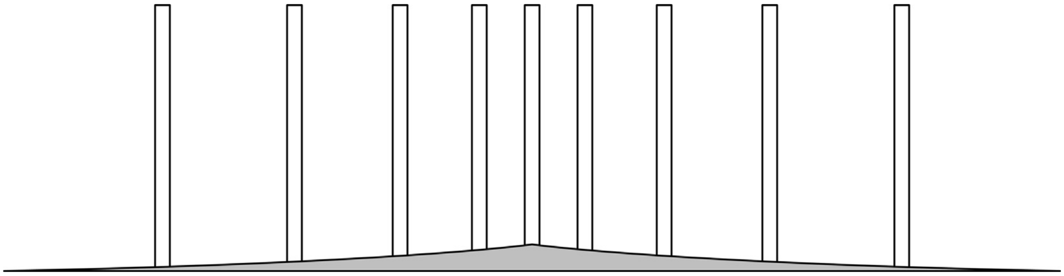

Fig. 2. Conductive base with identical fins distributed non uniformly.