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An iterative finite-element algorithm for solving two-dimensional nonlinear inverse heat conduction problems

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ABSTRACT

It is often useful to determine temperature and heat flux in multidimensional solid domains of arbitrary shape with inaccessible boundaries. In this study, an effective algorithm for solving boundary inverse heat conduction problems (IHCPs) is implemented: transient temperatures on inaccessible boundaries are estimated from redundant simulated measurements on accessible boundaries. A nonlinear heat equation is considered, where some of the material properties are dependent on temperature. The IHCP is reformulated as an optimization problem. The resulting functional is iteratively minimized using a conjugate gradient method together with an adjoint (dual) problem approach. The associated partial differential equations are solved using the finite-element package FEniCS. Tikhonov regularization is introduced to mitigate the ill-posedness of the IHCP. The accuracy of the implemented algorithm is assessed by comparing the solutions to the IHCP with the correct temperature values, on the inaccessible boundaries. The robustness of our method is tested by adding Gaussian noise to the initial conditions and redundant boundary data in the inverse problem formulation. A mesh independence study is performed.

1. Introduction

Inverse heat transfer problems (IHTPs) have raised increasing attention over the last decades thanks to their many scientific and technological applications. It is established that the IHTP is ill-posed in the sense of Hadamard ([6]); that is, the solution may not exist or be unique. Moreover, this problem is very sensitive to error, originating from input data, material properties, or rounding.

Based on the causal characteristics to be evaluated, IHTPs fall into four major classes: boundary, coefficient, retrospective, and geometric problems [3].

IHTPs can also be classified based on the heat transfer mechanism: conduction, convection, radiation, phase change (melting or solidification), or a combination of them [38].

Discretization methods range from control volume [41,12], boundary element [25], and finite difference [15,35], to classic finite element [26,33,16], mixed finite element and differential quadrature methods [18], and finite element with Trefftz functions [20].

Techniques commonly used for solving IHTPs include whole domain and sequential regularization methods [44], global and sequential function specification techniques [6,34], gradient iterative methods [2,25,21], stochastic optimization techniques [43], Monte Carlo methods [22,37], methods comprising filtering techniques [26], and neural networks [30,5,15].

Concerning regularization, it is introduced to obtain a stable, well-posed problem by minimizing an objective function. In this respect, Tikhonov regularization [44], Alifanov's iterative regularization [3], generalized eigensystem methods [42], and energetic regularization [13] are viable choices.

Here, our goal is to estimate the transient heat flux and temperature on an unreachable boundary from the transient radial heat flux and temperature on a reachable boundary. Consequently, we are faced with a boundary inverse heat conduction problem (IHCP). This problem is solved on the cross section of a hollow cylinder, which models the control-rod stems of several Swedish boiling water reactors. Thermal fatigue cracking was detected there in 2008 [45]. In our experiments and simulations, temperatures were sampled on the above cylinder when subject to a temperature range of 216 K and a pressure of 7.2 MPa. High pressures and temperatures in our test section prevented the installation of a large number of thermocouples to measure temperatures on all boundaries. Furthermore, as stressed in Meresse et al. [34], introducing more probes than necessary could perturb the thermal field in

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Nomenclature

Acronyms
CGM conjugate gradient method
DHCP direct (forward) heat conduction problem
IHCP inverse heat conduction problem

Greek symbols
α optimal step size
β conjugation coefficient
γ regularization parameter
Γ boundary
Γ_j boundary where q_j is prescribed
Δq_u direction in which q_u is perturbed
ε instantaneous error
η time-integrated error
θ solution to the sensitivity problem
ϑ angle
λ thermal conductivity
μ mean of the normal distribution
ξ standard deviation of the Gaussian noise
ρ density
σ standard deviation of the normal distribution
ϕ solution to the dual problem
Ω spatial domain

Roman symbols
c specific heat
h convective heat transfer coefficient
$\mathcal{J}$ objective function
k iteration number
$\mathbf{n}$ outward-pointing unit normal vector
p conjugate search direction
q_j heat flux
t time
t_f final integration time
$t_{\max}$ end time
T solution to the primal problem
$\mathcal{T}$ time domain $[0, t_{\max}]$
$\mathbf{x}$ spatial coordinate
Y noise-free solution to the DHCP
$\widetilde{Y}$ initial noise-free temperature
Y_∞ temperature of the medium

Subscripts
g related to the known heat flux
u related to the unknown heat flux

the solid domain. Algorithms accurately computing the temperature field in the above cylinder and other similar structures can help to validate computational fluid dynamics simulations more thoroughly. Moreover, thermal stresses can be computed from the solutions to 3D transient IHCPs. Here, we briefly review recent work on 2D and 3D transient IHCPs: in this article, we focus on 2D domains, but we plan to extend our approach to 3D domains of arbitrary geometry.

In Cebula and Taler [12], a space marching method is applied to reconstruct the transient heat flux on the outer surface of a hollow cylinder from temperature measurements inside the wall. Gram's polynomials smooth the input temperature time series to increase the robustness of the algorithm to measurement noise. The results are verified against CFD predictions.

In Mohammadiun [35], the conjugate gradient method (CGM) with adjoint equation is adopted to estimate the heat flux on the outer surface of a three-layer hollow cylinder from temperatures sampled at one point in the domain. Inner and side surfaces are insulated. The governing equations are discretized with finite-difference methods. No regularization is introduced because the proposed method is stable to measurement noise. The results are verified by numerical examples. A mesh independence test is performed.

In Guo et al. [21], temperatures on the inner wall of a horizontal T-junction with mixing flows are found from those on the outer wall, which is insulated, by performing least-squares optimization. Conjugate gradient and steepest descent methods are combined to solve the IHCP. A finite difference method is applied to solve the direct (forward) heat conduction problem (DHCP). Singular spectrum analysis is used to denoise the input temperature histories. Mesh and time-step independence tests are performed.

In spite of the many attempts to properly address boundary IHCPs, producing accurate, stable solutions to these problems is still a demanding challenge, especially if transients are fast and heat fluxes are highly varying in space [37]. Consequently, it is desirable to devise a theoretically well-grounded approach, which could provide reliable results in the presence of experimental noise, handle 3D geometries, and include temperature-dependent material properties, with no *a priori* information about the functional forms of the unknown boundary conditions.

In this work, we tried to do so by using a hybrid optimization algorithm, which employs the nonlinear CGM combined with an adjoint equation. A finite element discretization is adopted to solve the primal, dual, and sensitivity sub-problems. The problem is stabilized by Tikhonov regularization.

This article is structured as follows: the method employed is detailed in Section 2. Some results are shown in Sections 3.1 and 3.2. In the former case, temperatures on the inner boundary of an annulus are estimated from data on its outer boundary, whereas in the latter case, temperatures on the outer boundary of an annulus are estimated from data on its inner boundary. The relevance of the stopping criterion is stressed in the former case. The mesh independence of the results is assessed in the latter case. Some equations from Section 2 are proved in Appendices A and B.

2. Method

Two cases are considered: Test Problems 1 and 2 – see Fig. 1. In both cases, our inverse problem is solved by overspecifying the boundary conditions on Γ_g: in the inverse problem, the temperature time series on boundary Γ_u as well as the heat flux time series on boundary Γ_u are assumed to be unknown, whereas the initial temperature distribution, the temperature time series on boundary Γ_g as well as the heat flux time series on boundary Γ_g are assumed to be known. The former and latter heat flux time series are termed $q_u(\mathbf{x}, t)$ and $q_g(\mathbf{x}, t)$, respectively.

The inverse problem can be thought of as the minimization of the following objective function:

$$\mathcal{J}(q_{u,k}) = \frac{1}{2}\int_0^{t_f}\int_{\Gamma_g}\left(T_k - Y^{\xi}\right)^2 d\Gamma\, dt + \frac{\gamma_k}{2}\int_0^{t_f}\int_{\Gamma_u}\left(q_{u,k}\right)^2 d\Gamma\, dt. \tag{1}$$