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Numerical simulation of thermal flow of power-law fluids using lattice Boltzmann method on non-orthogonal grids

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ABSTRACT

In this study, a lattice Boltzmann model is proposed to investigate thermal flow of power-law fluids in irregular geometries. Non-orthogonal grids on the physical plane are adopted to accurately depict curved boundaries. With respect to power-law fluid, the evolutions of particle distribution functions for velocity and temperature fields on the computational plane are derived on the basis of the generalized form of interpolation-supplemented lattice Boltzmann method and the rheological equation. For forced convection, the double-distribution-function method is used to predict the temperature field. The coupled lattice BGK model is employed to consider the conditions under which the velocity and temperature fields are strongly coupled, such as natural convection. The adiabatic boundary and symmetry boundary in body-fitted coordinates are specifically prescribed. Numerical procedure is further validated by modeling a series of cases including forced convection over a stationary heated circular cylinder and natural convection in cavities and an annulus between concentric circular cylinders, respectively. Good agreements with available data in the literature are achieved. The numerical results demonstrate that the effect of rheological and thermodynamic properties of power-law fluid on the heat transfer performance and flow field in complex geometries can be suitably captured by present thermal lattice Boltzmann method on non-orthogonal grids.

1. Introduction

Forced and natural convections have received considerable attention due to their importance in engineering applications, such as electronic cooling systems, nuclear reactors, MEMS devices, solar energy collectors, and heat exchangers [1]. A large body of literature are focused on forced convection [2–5] and natural convection [6–11] of Newtonian fluids. However, in many practical applications such as petroleum drilling, pulp paper, food processing, geophysical system, and polymer engineering, the fluid exhibits complex non-Newtonian behavior. Considering the effect of rheological properties of non-Newtonian fluid on the hydrodynamics and thermodynamics, great efforts have been made on the studies of flow characteristics and heat transfer performances of non-Newtonian fluids.

A theoretical analysis of steady laminar natural convection heat transfer to power-law fluids was presented by Acrivos [12]. The well-established expression for the heat transfer rate to Newtonian fluid was generalized to include the non-Newtonian effects. Pittaman et al. [13] performed an experimental investigation on laminar natural convection of shear-thinning polymer solutions from an electrically-heated vertical plate, and obtained the dimensionless relation between Nusselt, Grashof and Prandtl numbers under conditions of constant surface heat flux. Recently, the nuclear Magnetic Resonance Imaging (MRI) technique was applied to study Rayleigh-Bénard convection of shear-thinning fluids in a cylindrical cavity. Compared with the Newtonian case, it is found out that the intensity of convection is increased by using shear-thinning fluids [14]. Numerical simulation is a complementary research approach to experimental and theoretical studies. Various numerical methods were applied to study natural convection of non-Newtonian fluids in past decades. The first investigation was attributed to Ozoe and Churchill, who developed a finite difference algorithm to study natural convection of a power-law fluid in a rectangle heated from below [15]. Later, this method was applied to investigate natural convection of power-law fluids in various enclosures, such as vertical rectangular cavity [16], square cavity [17], shallow cavity [18], and inclined rectangular enclosure [19]. Barth and Carey [20] reported a benchmark solution for natural convection of a shear-thinning fluid described by Powell-Eyring model in a cubical cavity using the finite element method. The

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finite volume method was also employed to study non-Newtonian heat transfer problems [21–26]. Turan et al. [21] investigated steady two-dimensional natural convection of power-law fluids in a square enclosure with differentially heated side wall subjected to constant temperatures. A new correlation was proposed for the mean Nusselt number in response to variations of Rayleigh number, Prandtl number, and the power-law index. Khezzar et al. [22] studied the effects of inclination angle and aspect ratio on natural convection of power-law type fluids in a two-dimensional rectangular tilted enclosure. Matin et al. [23] discussed the dependence of Nusselt number on eccentricity and aspect ratio for natural convection of power-law fluids between eccentric horizontal square ducts. These studies were focused on the cases with constant temperature boundary. Turan et al. [24] analyzed the influence of constant wall heat flux on laminar natural convection of power-law fluids in a rectangular enclosure. For transition buoyant convection of power-law fluids which is beyond the scope of present work, readers can refer to the work of Kim et al. [25] and references therein.

The heat transfer performances of power-law fluids in regular enclosures have been widely studied. However, the flow domain can be a complex geometry for most practical applications. For example, the wavy surfaces are beneficial to heat transfer rate [1]. Under such circumstances, the traditional numerical simulation methods, such as finite difference, finite volume, and finite element methods, involve tedious grid generation and complicated solution process [27]. In terms of numerical approaches for fluid flow in complex geometry, it can be divided into three categories: the unstructured mesh technique, the immersed boundary method on Cartesian grids, and the boundary-fitted curvilinear method [28]. The unstructured mesh technique can have a different number of neighbors, so it has a great flexibility in adapting the local grids to the arbitrarily shaped boundaries [9]. Bharti et al. [29] numerically investigated the heat transfer from an unconfined elliptical cylinder by using the finite volume method on unstructured non-uniform grids to yield the cylinder surface. Sasmal and Chhabra [30] adopted the unstructured cells with non-uniform spacing to study laminar natural convection from a heated square cylinder immersed in power-law liquids. The immersed boundary method was firstly proposed by Peskin [31] to analyze the blood flow patterns around heart valves. In terms of irregular boundaries, a body force was introduced in the momentum equation on Cartesian grids to avoid the generation of body-fitted grids. Ren et al. [32,33] applied this method to simulate thermal flow of Newtonian fluid with Dirichlet and Neumann temperature conditions, respectively. Zhang et al. [34] developed an immersed boundary method to study forced convection of Newtonian fluid over a circular cylinder. In the third methodology, the grids in physical domain are non-orthogonal to coincide with irregular boundaries. The governing equations can be solved numerically by the finite difference method [35] and the finite volume method [36] in the computational domain with uniform grids. Thus, the grid generation and transformation of governing equations from the physical domain to the computational domain are required. Some interesting results on natural convection of power-law fluids can be found in the literature [37,38].

In recent years, the lattice Boltzmann method (LBM), which is viewed as a particle-based numerical approach at mesoscopic level, has become a powerful approach for modelling isothermal and non-isothermal fluid flows. Different from the Navier-Stokes solvers, the LBM is based on kinetic theory. This feature provides a way to include microscopic dynamics at fluid-fluid and fluid-solid interfaces. Due to particulate nature and local dynamics, the LBM has many attractive advantages such as the ability of incorporating microscopic interactions, the capability of handling complex geometry, the simplicity of programming and the parallel algorithm [39]. As for application of the LBM in non-Newtonian fluid flows, first attempt was attributed to Aharonov and Rothman [40]. They proposed a new lattice Boltzmann model for power-law fluids in 1993. They allowed the local value of the viscosity at each lattice node to be dependent on strain rate tensor, consequently the dimensionless relaxation time at each node can be determined. Since then, the non-Newtonian LBM models for Bingham fluid [41], Herschel-Bulkley fluid [42], and viscoelastic fluid [43] have been developed successively. Particularly, Boyd et al. [44] pointed out that the strain rate tensor can be calculated locally at each lattice node in the LBM, which made this scheme efficient by avoiding the calculation of velocity derivatives. Nazari et al. [45] explored the thermal behavior of power-law fluids in a channel with a built-in porous square cylinder by using the lattice Boltzmann method. Delouei et al. [46] studied the unconfined flow and heat transfer of a power-law fluid over a heated cylinder by an immersed boundary-thermal lattice Boltzmann method. Additionally, the finite difference lattice Boltzmann method (FDLBM) was adopted to simulate natural convection of non-Newtonian molten polymer in sinusoidal heated cavity [47].

In the framework of LBM, the adaptive meshes refinement method and inter/extrapolated curved boundary method were presented to deal with curved boundaries [48–51]. Recently, Chen et al. [52] used the curved and moving boundary methods to simulate convective heat transfer of power-law fluids in an impeller stirred tank involving complex moving boundaries. All these techniques were performed on uniform grids. In addition, the interpolation-supplemented LBM (ISLBM) on non-uniform grids has been proved to be efficient in comparison with the conventional LBM for the simulation of fluid flow in complex geometries [53]. Based on irregular girds, the particles at nodes cannot move to their neighbors, so an interpolation step is introduced to determine the particle distribution functions at the lattice nodes after the streaming steps. It is noted that an analytical function is required to describe the transformation between the physical plane and the computational plane, which hampers its further application. Inspired by grid generation in body-fitted coordinates, Imamura et al. [54,55] developed the generalized form of interpolation-supplemented LBM (GILBM) and successfully applied it to simulate flow around a circular cylinder and NACA0012 airfoil, respectively. The GILBM breaks through the limitation of ISLBM and the computational efficiency is improved as well [56]. Later, Li et al. [57] developed a 2D multi-relaxation-time (MRT) lattice Boltzmann model on non-uniform mesh to study fluid flow in configurations of curved boundary. Mirzaei and Poozesh [58] also developed an interpolation lattice Boltzmann method using Joukowski transformation to link the computational plane and the physical plane, and applied it to study the 2D fluid flow around an airfoil in body-fitted coordinates.

Based on the foregoing analysis, it is evident that although the lattice Boltzmann method on non-uniform grids has been applied to hydrodynamics, very few attempts have been made to solve the energy equation, let alone the heat transfer problem involving non-Newtonian fluids. Li et al. [59] proposed a thermal LBM model on non-orthogonal grids for Newtonian fluid and validated it by simulating flow around a circular cylinder and natural convection in horizontal concentric and eccentric cylindrical annuli, respectively. The purpose of this article is to develop a thermal LBM model on non-uniform grids for heat-transfer application of power-law fluids. The present method adapts the GILBM model for the fluid motion in body-fitted coordinates. In terms of thermal flow, the double-distribution-function method proposed by Guo et al. [60] is introduced to consider the temperature evolution. The adiabatic boundary and symmetry boundary in body-fitted coordinates are derived. The accuracy of the simulation method is verified by simulating forced convection over a stationary