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An investigation of non-equilibrium heat transport in a gas system under external force field

Tianbai Xiao [a], Kun Xu [b,c,*], Qingdong Cai [a], Tiezheng Qian [b]

[a] Department of Mechanics and Engineering Science, College of Engineering, Peking University, Beijing 100871, China
[b] Department of Mathematics, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong
[c] Department of Mechanical and Aerospace Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong

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ABSTRACT

The gas dynamics under external force field is essentially associated with multiple scale nature due to the large variations of density and local Knudsen number. Single scale governing equations, such as the Boltzmann and Navier-Stokes equations, are valid in their respective modeling scales. Without identifying a physical scale between the above two limits for the modeling of the flow motion, it is challenging to develop a multiple scale method to capture non-equilibrium flow physics seamlessly across all regimes. Based on the modeling scale of cell size and implementing conservation laws directly in a discretized space, a well-balanced unified gas-kinetic scheme (UGKS) for multiscale gaseous flow has been constructed and used in the study of non-equilibrium flow and heat transport under external force field. In this paper, static heat conduction problems under external force field in different flow regimes are quantitatively investigated. In the lid-driven cavity case, the stratified flow is observed under external force field. With the increment of external force, the flow topological structure changes dramatically, and the temperature gradient, shearing stress, and external force play different roles in the determination of the total heat flux in different layers corresponding to different flow regimes. As a typical non-Fourier's heat conduction phenomena in the transition regime, the external force enhances the heat flux significantly along the forcing direction, with the relationship $\vec{q}_{force} \propto \vec{\phi}$, where $\vec{q}_{force}$ is the force-induced heat flux and $\vec{\phi}$ is the external force acceleration. This relationship is valid in all flow regimes with non-vanishing viscosity coefficient or the limited length of particle mean free path. Both theoretical analysis and numerical experiments are used to show the important role of external force on non-equilibrium heat transfer.

1. Introduction

The gas dynamics under external force field is usually associated with multiple scale nature due to the possible large variation of gas density and local Knudsen number along the direction of force. On mesoscopic level, the kinetic theory could be employed to illustrate the physical effect of external force. In the kinetic scale, the Boltzmann equation follows the evolution of velocity distribution function $f(x_i, t, u_i)$ to describe the particle transport and collision assembly. With an external forcing acceleration ϕ_i acting on the particles, the evolving process of f is modeled in Eq. (1) with separate operators: the free flight of the particles (left hand terms) and their collisions (right hand term), i.e.,

$$\frac{\partial f}{\partial t} + u_i \frac{\partial f}{\partial x_i} + \phi_i \frac{\partial f}{\partial u_i} = Q(f). \tag{1}$$

Here u_i is the particle velocity and $Q(f)$ is the collision term. For a real gas dynamic system, even with an initial Maxwellian of a barometric distribution under external force, the free transport of particles between two successive collisions always evolves the system towards to a non-equilibrium state. Under external force field, the particle acceleration or deceleration process during this time interval results in a distortion of the distribution function in the velocity space. The deviation from equilibrium distribution is restricted by the particle collision time τ. On the other hand, the particle collision takes effect to push the system back towards equilibrium state. In continuum flow, the deviation from equilibrium is weak due to intensive intermolecular collisions, and thus the non-equilibrium transport is well described with viscosity and heat conductivity in the constitutive relationship. However, in transition and rarefied regime, the particle free transport and collision are loosely coupled

* Corresponding author at: Department of Mathematics, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong.

E-mail addresses: xiaotianbai@pku.edu.cn (T. Xiao), makxu@ust.hk (K. Xu), caiqd@pku.edu.cn (Q. Cai), maqian@ust.hk (T. Qian).

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due to a large particle collision time. Much complicated nonlinear dynamics due to external force on the movement of particles between collisions can emerge and present a peculiar non-conventional energy or heat conduction transport phenomena. The strong non-equilibrium effects are expected in an even highly dissipative regions, such as the shock and boundary layers. To investigate the heat conduction problem under the existence of external force field is important to understand the dissipative flow physics.

It is noted that there is only limited study on non-equilibrium flow under external force field [1,2]. Generally, the existence of external force field, such as gravity, introduces a characteristic length scale $H \sim k_B T/m\phi$ [3], where k_B is the Boltzmann constant, m is the particle mass and ϕ is the magnitude of external forcing acceleration. It denotes a length scale over which the force field produces a significant effect on the gas evolution. For a gravitational system on the Earth atmosphere H is of $O(10^3)$ meters, and under laboratory condition or a micro-electro-mechanical system (MEMS) with geometric characteristic length L, the relation $H \gg L$ holds naturally, and thus it is reasonable to omit the influence of external force effect. However, in the case where H is comparable to L, the effect of external force will appear. For a large-scale system in stellar and planetary atmosphere, the force will result in significant change of gas density, and so is the variation of the particle mean free path and the local Knudsen number. Similar cases may appear in small scale, but with large acceleration, such as material interface with shock impingement. It is interesting to study the multiple scale non-equilibrium transport under external force filed.

The current existing governing equations for the gas dynamics, such as the Boltzmann and Navier-Stokes, are constructed on their respective modeling scales. For example, the Boltzmann equation is defined on the particle mean free path and collision time. Only on such a modeling scale, the particle transport and collision can be separately formulated, and the solutions in other scales by applying the Boltzmann equation is mainly due to accumulation of flow dynamics in the mean free path and particle collision time scale. The Navier-Stokes-Fourier (NSF) equations are constructed to describe fluid motion and heat transfer on a macroscopic level with limited number of flow variables. The fluid element is the finest closed unit in the NS modeling, where intensive particle collisions prevent particle penetration between adjacent fluid elements. The successful applications of the NS and Boltzmann equations are on their valid scales with a clear scale separation. However, for a system under external force field, the flow physics may vary continuously from the kinetic Boltzmann modeling in the upper rarefied layer to the hydrodynamic one in the lower dense region. The continuous variation of characteristic scale length should associate with a continuum spectrum of gas dynamics to connect the Boltzmann and NS seamlessly. However, it is challenging to construct such a multiple scale governing equation with flexible degrees of freedom to describe a scale-dependent dynamics, such as a general equation to in all scales from kinetic to hydrodynamic ones. The mathematical derivation of extended hydrodynamic equations will not be very successful if a modeling scale is not specifically pointed out. Unfortunately, there is no a clear physical scale for modeling between the Boltzmann and NS limits.

For conventional research of gas dynamics, the modeling and computation are handled separately. Once the governing equations are given, the CFD method serves to get numerical solution of differential equations, such as direct Boltzmann solver [4] for the Boltzmann equation and Riemann solver [5] for macroscopic fluid dynamic equations. Without valid multiple scale governing equations, the traditional CFD can be hardly used for solving multiple scale flow problem. In order to construct a multiscale method, the physics modeling is directly used in the construction of a numerical algorithm. Based on the cell size and time step scales, the corresponding discretized multiscale governing equations have been constructed in the well-balanced unified gas-kinetic scheme (UGKS) for flow problem under external force field [6–8]. Through a coupled treatment of particle transport, collision, and external forcing effect in the mesh size scale for the flux transport across a cell interface, a cross-scale flow physics from kinetic particle transport to hydrodynamic wave propagation has been incorporated in the scheme [9]. In the current work, the well-balanced UGKS will be employed to investigate the non-equilibrium gas evolution under external force field.

In this paper, the heat transfer in the lid-driven cavity flow is used as a typical example for the study of non-equilibrium gas dynamics under external force field with a large variation of gas density. Even under such a simple geometry, the cavity flow displays complicated flow phenomena with multiple scale transport, including shearing layers, eddies, secondary flows, heat transfer, hydrodynamic instabilities, and laminar-turbulence transition, etc [10]. Great efforts have been devoted to the study of the flow physics in different flow regimes. In the continuum regime, the cavity problem is a typical benchmark case for the validation of numerical algorithms for the NS solutions [10–15]. In rarefied regime, the direct simulation Monte Carlo (DSMC) [16] and Boltzmann solvers [17,18] provide the benchmark solutions. Naris et al. [19] discretized a linearized BGK equation to investigate the rarefaction effect on the flow pattern and dynamics over the whole range of the Knudsen number. Mizzi et al. [20] compared the simulation results from the Navier-Stokes-Fourier equations (NSF) with slip boundary conditions and the DSMC results in a lid-driven micro cavity case. John et al. [21] applied the DSMC, discovered counter-gradient heat transport in the transition regime, and investigated the dynamic effect from the expansion cooling and viscous heating on the heat transport mechanism. In all previous work, there is few study about the flow under external force field. Due to the external force effect, the cavity flow becomes even more complicated with its non-equilibrium multiple scale evolution. A few new phenomena, including the connection between the heat transfer and external force, and stratified flow of different regimes, have been observed through this study.

This paper is organized as follows. The basic kinetic theory and the analysis of external force on a gas dynamic system are presented in Section 2. Section 3 presents the numerical experiment and discussion on the non-equilibrium flow and heat transfer across different flow regimes. The last section is the conclusion.

2. Analysis on physical effect from external force

In the Chapman-Enskog expansion [22], the particle distribution function is expanded into series around the equilibrium state with respect to a small factor ϵ,

$$f = f^{(0)} + f^{(1)}\epsilon + f^{(2)}\epsilon^2 + \cdots, \tag{2}$$

and different truncations correspond to different fluid dynamic equations. With zeroth-order approximation, the distribution function stays in the exact Maxwellian, and the vanishing contribution of collision operator $Q(f)$ in Eq. (1) leads to the Euler equations [23],

$$\begin{aligned}
&\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho U_i) = 0,\\
&\frac{\partial}{\partial t}(\rho U_i) + \frac{\partial}{\partial x_j}(\rho U_i U_j + p\delta_{ij}) = -\rho\phi_i,\\
&\frac{\partial}{\partial t}\left[\rho\left(e + \frac{1}{2}U_i U_i\right)\right] + \frac{\partial}{\partial x_j}\left[\rho U_j\left(e + \frac{1}{2}U_i U_i\right) + U_i p \delta_{ij}\right] = -\rho U_j \phi_j.
\end{aligned} \tag{3}$$