International Journal of Heat and Mass Transfer 126 (2018) 466–478

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

An analytical solution to the dynamic behavior of heat exchanger networks

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A R T I C L E I N F O

Article history:
Received 24 December 2017
Received in revised form 3 April 2018
Accepted 8 May 2018

Keywords:
Transient response
Dynamic simulation
Analytical solution
Heat exchanger networks (HEN)

A B S T R A C T

A novel method combined signal flow graph of a single heat exchanger with the transfer function of streams is developed for the dynamic behaviors of heat exchanger networks problems, which are determinate factors of the process control and operation optimization in the processing industries. The transfer functions between any two nodes of heat exchanger networks including the inlet and the outlet are obtained based on the signal flow graph of the networks by block-diagram reduction, Mason's rule and the seeking-up method. The developed method is solved by a numerical inverse Laplace transform and the analytical solution to the dynamic behavior of heat exchanger networks is presented in the time domain. The numerical results demonstrate that the presented method is more efficient and more accurate for the dynamic behaviors of heat exchanger networks problems.

1. Introduction

The dynamic behavior of heat exchanger networks (HEN) has been taken into consideration because its dynamic response of outlet parameters to the disturbances of inlet parameters is important for the process controllability and operation optimization of the HEN problems [1–3]. Nowadays, substantial numerical and experimental investigations have been carried out for the dynamic behavior of HEN, even including a single heat exchanger due to its wide applications [4–9]. For a single heat exchanger, many dynamic mathematics models [10–16] have been presented by scholars. The results showed that the transient temperature responses of streams can be obtained using analytic methods [17,18] or numerical methods [19,20].

For the HEN problems, the excessive simplifications of the dynamic mathematics model lead to low quality of dynamic simulations, whereas more consideration of the actual conditions in HEN would make the model complicated and increase computational cost. Therefore, the dynamic mathematics model is far from sufficient to apply to the dynamic characteristics of HEN with consideration into more factors [21]. Above all, achieving an efficient and high-accurate solution for the dynamic behavior of the HEN problems is more important than proposing a dynamic mathematics model which can be solved based on the modeling of single heat exchanger.

During the past few decades, many effective numerical methods benefiting from the progress of the dynamic modeling of HEN have been presented to investigate the dynamic behavior of HEN [22–29]. Mathisen et al. [22] proposed a dynamic model of relatively simple HEN taking the structure, pipe residence time, model order of bypasses and the connecting pipes into consideration. The dynamic model based on the lumped model for a single heat exchanger was solved using the state-space method for the dynamic behavior of HEN and was implemented in Simulink. It's shown that the presented dynamic model can be not for more complicated HEN owing to modeling complexity, computational speed, and numerical stability. Several dynamic models for HEN have been developed for the dynamic behavior of HEN problems in different practical requirements [23–26]. Boyaci et al. [27] used a distributed-parameter model consisting of multi-tube, single-pass heat exchangers to construct a dynamic model of the HEN and investigated numerically the transient behavior of the HEN. Baldea et al. [28] presented a procedure deriving reduced-order, non-stiff models, which was focused on the dynamics and control of HEN consisting of a reactor connected with an external heat exchanger through a large material recycle stream that acts as an energy carrier. Guha and Ghaudhuri [29] developed a mathematical model to describe the transient heat exchange between the process streams of HEN and solved the developed model by a finite difference numerical scheme and solution algorithm.

Although numerical methods can almost deal properly with the dynamic behavior of all types of HEN problems, the methods are invalid for real-time control. Moreover, higher computational

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https://doi.org/10.1016/j.ijheatmasstransfer.2018.05.041
0017-9310/© 2018 Published by Elsevier Ltd.

Nomenclature

A	area of heat exchanger, m^2
C_p	specific heat of the stream, kJ/(kg K)
G	mass flow rate, kg/s
K	gain
T	time constant
$U(s)$	inlet parameter of transfer function
v	velocity of stream, m/s
$W(s)$	operator transmittance
$Y(s)$	outlet parameter of transfer function
h	heat transfer coefficient, $W/(m^2 K)$
τ	delay time, s

Subscripts

c	cold stream
h	hot streams
i	inlet
o	outlet
q	heat
t	temperature
w	metal wall of heat exchanger

efforts and costs are required when the methods are implemented for more complicated and larger-scale HEN. Nowadays, analytical solution of HEN has drawn attentions from many researchers due to its high efficiency for dynamic behavior of HEN.

In present study, a novel method based on signal flow graphs is established to obtain an analytical solution for the dynamic behavior of HEN. First, a single heat exchanger is treated as a 4×4 Multiple-Input Multiple-Output (MIMO) system, and the signal flow graph of the HEN is established according to the transitive relation among streams. Second, three methods (block-diagram reduction, Mason's rule and seeking up) are used to obtain the transfer function from the inlet to outlet nodes or between any two random nodes. Finally, according to a numerical inverse Laplace transform, an analytical solution for the dynamic behavior of HEN is presented in the time domain.

2. Approaches

2.1. Transfer functions and signal flow graph for a single heat exchanger

For the HEN system, the outlet temperature response of a single heat exchanger can be approximated to transfer function of a first-order or second-order inertial element. In present study, the first-order model with time delay used to the simplification for the transient profile of heat exchangers [30,31] is employed to describe the dynamic behavior of the single heat exchanger. Therefore, There-fore, the ratio of output temperature $Y(s)$ to input disturbance $U(s)$ in the Laplace domain can be given as [32],

$$W(s) = \frac{Y(s)}{U(s)} = \frac{K}{Ts+1} e^{-\tau s} \tag{1}$$

where K, T and τ are shown in the Appendix A. The three dynamic parameters are obtained by the logarithmic mean temperature difference for higher static accuracy. Detailed derivation process of K, T and τ can be seen in the Sections 2-4 of previous research [33].

With similar to other heat-exchanger models in which the absence of fluid phase-change [22,34,35], the simplifying assumptions are following: (1) the heat-exchanging fluids are fully mixed and their temperatures are constant; (2) both cold and hot fluids are incompressible and their pressure drops are negligibly small, it is often appropriate to assume that delay effect only acts on the stream with inlet temperature disturbance, and time constant is identical for temperature and flow-rate disturbances; (3) thermal-physical parameters of both cold and hot fluids and tube-wall materials are constant; (4) heat conduction between units can be neglected; (5) heat accumulation in the shell wall material can be neglected; (6) the values of heat transfer coefficients determined for steady state conditions remain unchanged in transient states; (7) heat losses to the environment are negligibly small.

For a single heat exchanger shown in Fig. 1(a), there are four potential disturbances including flow disturbances and temperature disturbances of the cold and hot inlet streams. Therefore, the outlet temperature responses of cold and hot streams in single heat exchanger can be expressed as follows,

$$\Delta t_{co}(s) = W_{ctc}(s)\Delta t_{ci}(s) + W_{cth}(s)\Delta t_{hi}(s) + W_{cGc}(s)\Delta G_{ci}(s) + W_{cGh}(s)\Delta G_{hi}(s) \tag{2}$$

$$\Delta t_{ho}(s) = W_{hth}(s)\Delta t_{hi}(s) + W_{htc}(s)\Delta t_{ci}(s) + W_{hGc}(s)\Delta G_{ci}(s) + W_{hGh}(s)\Delta G_{hi}(s) \tag{3}$$

In addition to the outlet temperature responses, which are expressed as functions of all inlet disturbances, the responses of the outlet flow rate are also treated as functions of the inlet

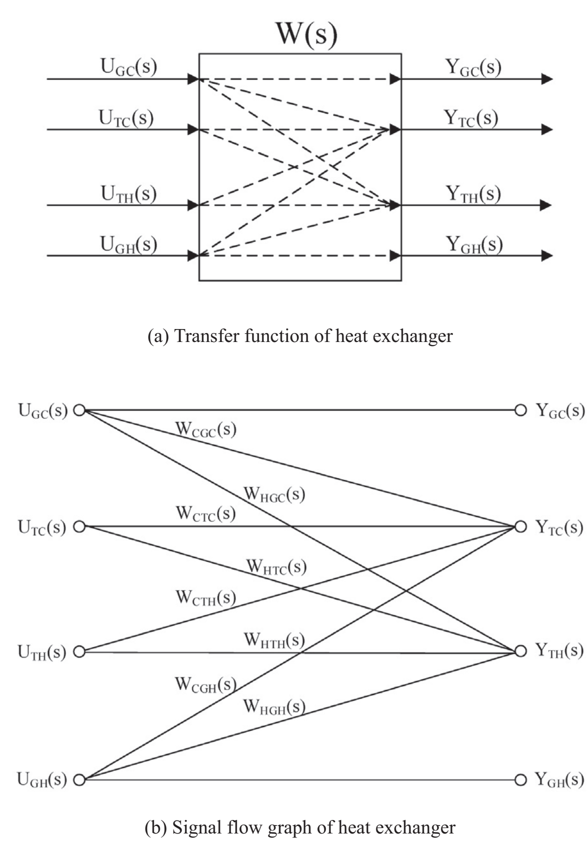

Fig. 1. Transfer function and signal flow graph of heat exchanger.