



## Bidispersive double diffusive convection

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### ABSTRACT

A model is developed for double diffusive convection in a bidisperse porous medium. Double diffusive convection is convective movement of fluid due to temperature and salt gradient effects. A bidisperse porous medium is one where there are pores known as macropores, but the solid skeleton contains cracks or fissures which give rise to a porosity in the skeleton, known as microporosity. We concentrate on the case of a single temperature field and attention is focussed on the situation where the layer is heated from below and simultaneously salted from below.

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### 1. Introduction

There is much current interest in the behaviour of double porosity, or bidispersive, porous materials. A double porosity material is one which possesses the normal pore structure, such pores being known as macropores, but, the solid skeleton has cracks or fissures in it and this gives rise to a second porosity, the smaller pores being referred to as micropores. Heat and mass transfer in a bidispersive porous material has been of interest in the chemical engineering field for some time now, see e.g. Burghardt et al. [1], Szczygiel [2–4], Valus & Schneider [5]. The interest in heat and mass transfer in a bidispersive porous material is driven by the many real practical applications of these phenomena, for example, to landslides, see e.g. Montrasio et al. [6], Borja et al. [7], Borja & White [8], Pooley [9] and Scotto di Santolo & Evangelista [10]. Another important research area for heat and mass transfer in a bidisperse porous medium is in biporous wicks in heat pipes, see e.g. Taqvi et al. [11], Lin et al. [12], Mottet & Prat [13], Yeh et al. [14]. A further mundane area involves stockpiling coal, Hooman & Maas [15], Hooman et al. [16]. Relatively small pieces of coal are stockpiled but the coal itself contains small pores. The porosity of the stockpile is usually higher than that in the solid coal, and the fact that these piles can self heat makes an analysis and understanding of heat transfer vital to prevent self combustion. There are many other applications of bidispersive porous media and lots of these are mentioned in the monograph by Straughan [17].

Thermosolutal porous convection, also known as double diffusive porous convection, involves fluid movement in a porous

medium in a non-isothermal situation where there is a salt dissolved in the fluid. In porous media this convection for a single porosity medium was analysed in the fundamental article of Nield [18]. Many subsequent articles have appeared dealing with linear instability, but also clever analyses of nonlinear stability, see e.g. Joseph [19,20], Barletta & Nield [21], Mulone [22], Love et al. [23], Simmons et al. [24], Deepika & Narayana [25], Deepika [26], Straughan [27]. The nonlinear stability aspect of thermosolutal convection in a porous medium from an energy method viewpoint is the focus of attention of Lombardo et al. [28], with further information being included in the books by Straughan [29,30].

Fundamental theories for thermal convection in a bidisperse porous medium were developed and analysed by Nield and Kuznetsov [31–35] and by Nield [36]. This work is reviewed in Straughan [30]. Falsaperla et al. [37] and Gentile & Straughan [38] continued analysis with the Nield-Kuznetsov models but they restrict attention to the case where only one temperature is present whereas Nield and Kuznetsov [32] allowed for different temperatures in the fluid in the macro and micro pores. Further work using the single temperature model has been given by Gentile & Straughan [39] and by Franchi et al. [40].

The aim of this paper is to present a model for fluid flow in a bidisperse porous medium which allows for thermosolutal convection, i.e. for convective motion incorporating temperature and salt effects. We employ a single temperature and we analyse in detail the problem of determining the onset of convective movement when the layer is heated from below. Equations are derived which allow for the layer to be salted below or above, although we concentrate on the more interesting, and more complicated, case of salted from below.

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We also incorporate the Soret effect, see e.g. Soret [41], Platten [42], Straughan [30], p. 40. The effect of a temperature gradient on mass transfer in a bidisperse porous medium may well be of much interest. We note that in the case of a single porosity body the problem of thermosolutal convection in a porous medium taking into account the Soret effect has been the topic of the recent article by Deepika [26]. The analysis of Deepika [26] effectively concentrates on the heated below - salted above case and so there is no overlap with the present work, which is also as far as we are aware the first analysis of double diffusive convection in a bidisperse porous medium.

**2. Governing equations**

Let  $\phi$  be the porosity associated to the macropores, so that  $\phi$  is the ratio of the volume of the macropores to the total volume of the saturated porous material. Let  $\epsilon$  be the porosity associated to the micropores, i.e.  $\epsilon$  is the ratio of the volume occupied by the micropores to the volume of the porous body which remains once the macropores are removed. Thus the fraction of volume occupied by the micropores is  $\epsilon(1 - \phi)$ .

We follow Nield and Kuznetsov [32] and use sub or superscript  $f$  and  $p$  to denote a quantity associated to the macropores or micropores, respectively. Let  $V_i^f$  and  $V_i^p$  be the pore averaged velocities in the macro and micropores. Then the analogous seepage velocities  $U_i^f$  and  $U_i^p$  are given by

$$U_i^f = \phi V_i^f, \quad U_i^p = \epsilon(1 - \phi) V_i^p.$$

We assume the density in the buoyancy force is a linear function of the temperature,  $T$ , and salt concentration,  $C$ , and so

$$\rho = \rho_0[1 - \alpha(T - T_0) + \alpha_c(C - C_0)]$$

where  $\alpha$  is the thermal expansion coefficient and  $\alpha_c$  is the equivalent expression for the salt field. Then, employing a Boussinesq approximation the momentum and continuity equations in the macropores and micropores are derived as in Nield and Kuznetsov [32], Falsaperla et al. [37] or Gentile & Straughan [38], and are

$$\begin{aligned} 0 &= -\frac{\mu}{K_f} U_i^f - p_i^f - \zeta(U_i^f - U_i^p) + g\rho_0\alpha k_i T - \alpha_c\rho_0 g k_i C, \\ U_{i,i}^f &= 0, \\ 0 &= -\frac{\mu}{K_p} U_i^p - p_i^p - \zeta(U_i^p - U_i^f) + g\rho_0\alpha k_i T - \alpha_c\rho_0 g k_i C, \\ U_{i,i}^p &= 0, \end{aligned} \tag{1}$$

where  $\mu$  is the dynamic viscosity of the fluid and  $\zeta$  is an interaction coefficient, namely the coefficient for momentum transfer between the macro and micro phases, see Nield & Kuznetsov [32]. The quantities  $p^f$  and  $p^p$  are the pressures in the macro and micro phases,  $\mathbf{k} = (0, 0, 1)$  and  $\mathbf{g}$  is the size of the gravity vector. The terms  $K_f$  and  $K_p$  are the permeabilities in the macro and micro phases. Throughout we employ standard indicial notation.

The equation governing the energy balance, i.e. the equation for the temperature field is derived from the equations of Nield & Kuznetsov [32] as in Gentile & Straughan [38] and has the form

$$(\rho c)_m T_{,t} + (\rho c)_f (U_i^f + U_i^p) T_{,i} = \kappa_m \Delta T, \tag{2}$$

where  $\rho$  is the density,  $c$  is the specific heat at constant pressure,  $f$  denotes the macro phase and

$$\begin{aligned} (\rho c)_m &= (1 - \epsilon)(1 - \phi)(\rho c)_s + [\phi + \epsilon(1 - \phi)](\rho c)_f, \\ \kappa_m &= (1 - \epsilon)(1 - \phi)\kappa_s + [\phi + \epsilon(1 - \phi)]\kappa_f, \end{aligned}$$

where  $(\rho c)_s, (\rho c)_f, \kappa_f$  and  $\kappa_s$  are the products of density and specific heat in the solid skeleton and in the fluid in the pores,

respectively, and the thermal conductivity of the fluid and solid, respectively.

We need to derive an equation for the concentration. To this end we note that when the diffusion coefficient includes the Soret effect it has form

$$\mathbf{J}_c = -k_c \nabla C - k_c S_T \nabla T$$

where  $\mathbf{J}_c$  is the diffusion coefficient and  $S_T$  is a Soret coefficient. We thus write differential equations for the macro and micro phases as

$$\frac{\partial C}{\partial t} + V_i^f \frac{\partial C}{\partial x_i} = k_c^f \Delta C + k_c^f S_T^f \Delta T, \tag{3}$$

and

$$\frac{\partial C}{\partial t} + V_i^p \frac{\partial C}{\partial x_i} = k_c^p \Delta C + k_c^p S_T^p \Delta T. \tag{4}$$

Multiply (3) by  $\phi$  and (4) by  $\epsilon(1 - \phi)$  and add the results. In this way we derive the following equation for the concentration field throughout the porous medium continuum,

$$\epsilon_1 \frac{\partial C}{\partial t} + (U_i^f + U_i^p) \frac{\partial C}{\partial x_i} = \epsilon_2 \Delta C + S \Delta T, \tag{5}$$

where

$$\begin{aligned} \epsilon_1 &= \phi + \epsilon(1 - \phi), \quad \epsilon_2 = \phi k_c^f + \epsilon(1 - \phi) k_c^p, \\ S &= \phi S_T^f + \epsilon(1 - \phi) S_T^p. \end{aligned}$$

Thus, the governing system of equations for double diffusion in a bidisperse porous medium consist of (1), (2) and (5) for the variables  $U_i^f, U_i^p, p^f, p^p, T$  and  $C$ .

**3. Basic solution and perturbation equations**

We now investigate the problem of thermosolutal convection in a plane layer of bidisperse material. Let the saturated porous material occupy the horizontal layer  $0 < z < d, \{(x, y) \in \mathbb{R}^2\}$  and satisfy Eqs. (1), (2) and (5). The boundary conditions are,

$$\begin{aligned} U_i^f n_i &= 0, \quad U_i^p n_i = 0, \quad \text{on } z = 0, d, \\ T &= T_L, z = 0, \quad T = T_U, z = d, \\ C &= C_L, z = 0, \quad C = C_U, z = d, \end{aligned} \tag{6}$$

where  $n_i$  is the unit outward normal to the planes  $z = 0$  and  $z = d, T_L, T_U, C_L, C_U$  are constants with  $T_L > T_U$ . We derive the perturbation equations from the steady solution under the boundary conditions

$$C_L > C_U, \quad \text{salted below,} \tag{7}$$

or

$$C_L < C_U, \quad \text{salted above.} \tag{8}$$

The basic conduction solution is then

$$\bar{U}_i^f \equiv 0, \quad \bar{U}_i^p \equiv 0, \quad \bar{T} = T_L - \beta z, \quad \bar{C} = C_L - \beta_c z, \tag{9}$$

where

$$\beta = \frac{T_L - T_U}{d} > 0, \quad \beta_c = \frac{C_L - C_U}{d}. \tag{10}$$

When the layer is salted below  $\beta_c > 0$  whereas when the layer is salted above  $\beta_c < 0$ .

Let  $u_i^f, u_i^p, \pi^f, \pi^p, \theta, \gamma$  be a perturbation to the steady solution (9). The perturbations are non-dimensionalized with length scale  $D$ , time scale  $\mathcal{T}$ , velocity scale  $U$ , temperature scale  $T^\#$ , concentration scale  $C^\#$ , where

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