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Liquid-vapor flows in heated ducts: Reference solutions

M. Fathalli [a], F. Petitpas [a,*], S. Le Martelot [b]

[a] Aix Marseille Univ, CNRS, IUSTI, Marseille, France
[b] Centre National d'Etudes Spatiales, France

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ABSTRACT

Analytical one-dimensional steady state solutions in a heated duct of constant area are determined for two-phase liquid-gas and liquid-vapor mixtures. First, the exact solution for single-phase flow is recalled in the different flow regimes (subsonic and supersonic) for given inflow conditions. These solutions are extended in the context of duct connected to an upstream tank and to a downstream pressure outflow condition. Then, reference solutions for various two-phase flow models are developed for compressible two-phase flows. They are applied in the context of ideal gas and stiffened gas equation of state for liquid-gas and liquid-vapor flows. These solutions corresponds to limit situations of partial equilibrium among the phases. The first limit situation corresponds to a two-phase flow model where both phases evolve in mechanical equilibrium only. The second one corresponds to a two-phase model where the phases evolve in both mechanical and thermal equilibria. The last one corresponds to a model describing a liquid-vapor mixture in thermodynamical equilibrium. The three limit situations are compared to show the impact given the choice of one or another equilibrium on fluid state and in particular on mass flow rate in the duct.

1. Introduction

The modeling of liquid-vapor flows has been widely improved in past years [1–4]. Today, efficient models and numerical methods are today able to perform realistic and quantitative numerical simulations for two phase flows with heat transfers [5], phase transitions [6], droplet breakup with capillary effects [7], etc. Nevertheless, the simulations that take into account each of these effects together are quite confidential and often limited to a restrictive geometry (a droplet, a key element of an engine). Whatever the efficiency of the code is, this does not allow to simulate flows in extended geometry and in these cases, models and methods coupling that alternates between numerical and approximate solutions are used. This last remark is true in particular for flows in cryogenic spatial engines. Cryogenic fluids are flowing from tanks to combustion chamber passing through important length of ducts submitted to intense heat fluxes (from combustion chamber and nozzle). In most of the engine, the fluids are supercritical, therefore, phase change does not occur and the mass flow rate is well known thanks to non compressible assumptions. If the heat flux become very intense, phase change could occur and lead to vapor apparitions producing sonic lock and consequently decreases mass flow rates delivered to the combustion chamber. In this case, it is important to apprehend the flow along the heated duct in order to prevent difficulties. This kind of flows also appears during the cooling of pipes preceding ignition or re-ignition. In that case, liquid hydrogen (at 20 K) for example is flowing in a pipe at ambient temperature (300 K) or more. The cooling process is complex, not only using fluids at very low temperatures but also because of the geometry of the various cooling channels. These channels are shown in Fig. 1 which shows how the fluids, oxygen and hydrogen, circulate within the engine.

The aim of this work is to determine simplified analytical solutions allowing to evaluate the flow state variables for a two phase flow in a constant section duct submitted to important heat fluxes. These solutions will be determined in limit cases of homogeneous two phase flows. The study of heat transfers in heated gas ducts is well known (see for example [8]). Analytic solutions are available when the inflow conditions are known and imposed but this corresponds to simplified cases without coupling to an upstream flow as for example those obtain from a tank. For homogeneous liquid-vapor flows, numerical methods are proposed to compute approximate 1D solutions for steady and unsteady flows in [9]. Such compressible multiphase flows are complex to apprehend: On one hand, heat and mass transfers have a significant influence on the flow conditions (velocity, pressure, Mach number). Consequently,

* Corresponding author.
E-mail addresses: fathallimaroua@gmail.com (M. Fathalli), fabien.petitpas@univ-amu.fr (F. Petitpas), sebastien.lemartelot@cnes.fr (S. Le Martelot).

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Nomenclature

General quantities

$\dot{m}$	mass flow rate kg/s
$\dot{m}_s$	specific mass flow rate kg/(s m^2)
γ	heat capacity ratio
$\mathbf{I}$	identity matrix
ρ	density kg/m^3
υ	specific volume m^3/kg
c	speed of sound
C_p	specific heat capacity at constant pressure J/(K kg)
C_v	specific heat capacity at constant volume J/(K kg)
E	total specific energy J/kg
e	specific internal energy J/kg
g	specific Gibbs free energy J/kg
H	total specific enthalpy J/kg
h	specific enthalpy J/kg
M	Mach number
p	pressure Pa
p_∞	parameter for the "stiffened-gas" equation of state Pa
q	specific heat J/kg
r	specific gas constant J/(K kg)
s	specific entropy J/(K kg)
T	temperature K
u	velocity m/s

Phasic quantities

α_k	volume fraction of phase k
Y_k	mass fraction of phase k

they have a potential for nuisance on circulating flows in pipes when coupling with a tank, similar to the case of an adiabatic pipe with cross-sectional variation [10,11]. On the other hand, the topology of the flow is unknown and strongly conditioned by heat exchanges at walls and between phases. That leads to the need of extra closure relations (or engineering models) to compute exchange terms.

Unfortunately, and up to our knowledge, no analytic reference solutions can be found in literature for compressible multiphase flows, even in limit cases. This is also true when a connection between the duct and a tank is considered.

The paper is organized as follows: The first part is dedicated to single phase gas flows in heated ducts. We first recall theoretical bases of the analytical calculation of steady-state solutions in non-adiabatic duct for gas flows. Then new reference solutions with tank upstream inlet conditions are proposed. In the second part, a threefold extension of this approach to the two-phase flows is proposed:

- The two phases evolve in a mechanical equilibrium (equilibrium of pressures and velocities).
- The two phases evolve in a mechanical and thermal equilibrium (equilibrium of the pressures, the speeds and the temperatures).
- The two phases evolve in thermodynamic equilibrium (equilibrium of pressures, speeds, temperatures and chemical potentials).

For all this cases, analytical or pseudo-analytical solutions are proposed. These solutions allow to easily evaluate the conditions along a 1D pipe subjected to a heat flux in limit cases. These three proposed limit flows also permit to study the impact of upstream flow condition modifications on the duct flow. Impact of equlbria on the flow is highlighted in a third part. The proposed solutions could be used as reference solutions for the validation of more sophisticated multi-dimensional simulation tools.

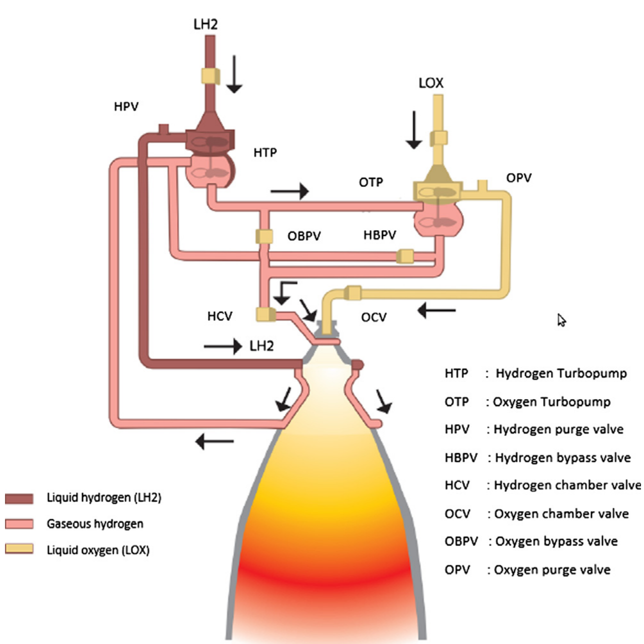

Fig. 1. Scheme representing the flow of fluids in ducts in a rocket engine.

2. Single phase flows in heated ducts

Let us consider a one-dimensional flow in a pipe of constant cross section S and of length L receiving a linear heat flux φ (W/m). The quantity of energy received per mass unit of the fluid is equal to $q = \frac{\varphi L}{\dot{m}}$ (J/kg) with $\dot{m}$ the mass flow rate of fluid in the duct. This configuration is presented in Fig. 2.

The inlet state is denoted by subscript "*in*" while the outlet state is denoted by subscript "*out*". For compressible inviscid single phase flows, the equations linking the inlet and the outlet fluid state (p, T, u) in the duct are the Euler equations:

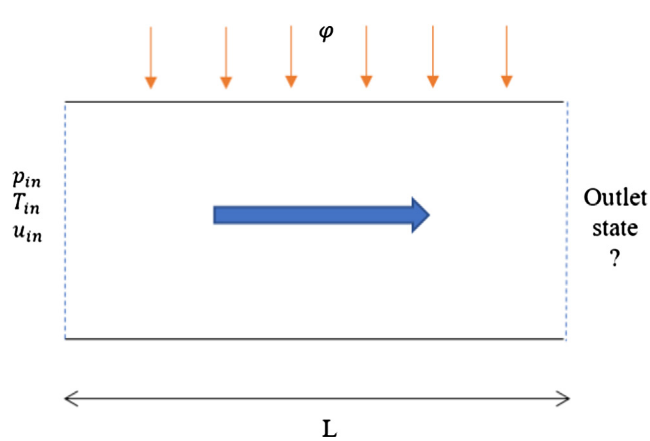

Fig. 2. Flow in a duct receiving a heat flux.