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Thermo-hydrodynamics of a viscoelastic fluid under asymmetrical heating

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ABSTRACT

We investigate the influence of viscous dissipation on the convective heat transfer and entropy generation characteristics for the Poiseuille flow of a viscoelastic fluid in an asymmetrically heated slit microchannel. We use the simplified Phan –Thein –Tanner constitutive equations to describe the rheology of the fluid. Considering hydrodynamic slip at fluid–solid interface, we analytically solve the transport equations to obtain the velocity and temperature fields inside the flow domain, which are further used to evaluate the Nusselt number and the thermodynamic irreversibility generation rate. The results show that the present thermo-fluidic transport process is strongly influenced by the governing dimensionless parameters *viz.*, the Brinkman number, heat flux ratio, viscoelastic group and slip coefficient. Finally, this study reveals that the Brinkman number and viscoelastic parameter have reverse effects on the Nusselt number and entropy generation rate as compared to the slip coefficient.

1. Introduction

Over the last few decades, the increasing trend of system miniaturization has led to the fabrication of micro devices that involve both heat transfer and fluid flow, e.g. micro heat exchanger, micro reactor, micro heat pipes etc. The extensive engineering applications of these thermal management systems in different areas have attracted the research community to explore the underlying microscale thermo-hydrodynamics. An increased surface area to volume ratio significantly affects the flow dynamics in such miniaturized systems. Two important aspects of microscale convection process, especially for non-Newtonian fluids, are the interfacial hydrodynamic slip and the viscous dissipation effect [1–5]. Noteworthily, while the hydrodynamic slip changes the flow dynamics significantly, the viscous dissipation effect markedly influences the convective heat transfer characteristics of the underlying transport process as reported in the literature [6–9]. It may be mentioned that the investigation of convective transport of heat in miniaturized systems/devices is one of the most fundamental and important problems in the paradigm of microscale heat transfer, since it forms the basis of the analysis of thermodynamic irreversibility in such systems/devices.

The viscoelastic fluids, such as polymer solutions, bio-fluids, gels, etc. are a group of non-Newtonian fluids that possesses both the viscous and elastic character. The stretching and scrolling of the long molecular chains give rise to the elastic nature of such fluids [10]. To imitate the flow behavior of viscoelastic fluids, several models *viz.* Phan-Thien-Tanner, Giesekus, Oldroyd-B, Maxwell model are available in the literature [11–14]. Albeit these models describe the rheology of viscoelastic fluids, nevertheless, the most conventional form of constitutive relationships are presented by the Phan –Thien –Tanner (PTT) model, which is derived from the network theory [11,15]. It is worth noting that considering this model, quite a few studies on viscoelastic fluids are available in the literature as well [16–22].

Thermal management systems/devices need to sustain different thermal boundary conditions accounting their wide applications in diversified fields. Typically, these include the uniform heat flux, uniform wall temperature or insulated wall boundary conditions. Numerous studies have been reported in the literature on analyzing the convective heat transport process under such boundary conditions [23–36]. However, depending upon the flow environment, sometimes an asymmetric thermal boundary condition also needs to be considered to analyze the underlying thermal

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Nomenclature

$a_1 - a_5$	coefficients defined in Appendix A
A_c	cross sectional area of channel, m^2
b	width of channel wall, m
c	constant of integration for velocity profile
c_p	specific heat capacity of fluid, J kg^{-1} K^{-1}
$d_1 - d_{10}$	coefficients defined in Appendix B
H	half channel height, m
$\hbar$	convective heat transfer coefficient, W m^{-2} K^{-1}
k_f	thermal conductivity of fluid, W m^{-1} K^{-1}
L	length of channel, m
k	slip coefficient, $(\mathrm{m\,s^{-1}})^{1-m}\,(\mathrm{Pa\,s\,m^{-1}})^{-m}$
$\bar{k}$	dimensionless slip coefficient, $kU^{m-1}(\eta/H)^m$
m	slip exponent
p	pressure, Pa
q	wall heat flux, W m^{-2}
q_r	wall heat flux ratio, q_2/q_1
$\mathbb{R}$	real number
S_G	local entropy generation rate, W m^{-3} K^{-1}
$\dot{S}$	dimensionless local entropy generation rate, $H^2 S_G/k_f$
$<\dot{S}>$	dimensionless global entropy generation rate
T	temperature, K
T_b	bulk mean fluid temperature, K
u	axial fluid velocity, m s^{-1}
$\bar{u}$	dimensionless velocity, u/U
x	axial coordinate, m
$\bar{x}$	dimensionless axial coordinate, x/H
y	transversal coordinate, m
$\bar{y}$	dimensionless transversal coordinate, y/H

Greek symbols

β	parameter defined in Eq. (23)
ρ	fluid density, kg m^{-3}
ε	extensibility parameter
η	viscosity coefficient, kg m^{-1} s^{-1}
λ	relaxation time, s
$\boldsymbol{\tau}$	stress tensor
$\underset{\sim}{\tau}$	component of stress tensor, Pa
$\overset{\nabla}{\boldsymbol{\tau}}$	upper convective derivative
θ	dimensionless temperature, defined in Eq. (21)
θ_b	dimensionless bulk mean temperature, $k_f(T_b - T_1)/q_1 H$
ψ	dimensionless upper wall heat flux, $q_1 H/k_f T_1$

Non dimensional numbers

Br	Brinkman number, $Br = \eta U^2/q_1 H$
De	Deborah number, $De = \lambda U/H$
Pe	Péclet number, $Pe = \rho c_p U H/k_f$
Nu	Nusselt number, $Nu = \hbar H/k_f$
Be	Bejan number, defined in Eq. (32)

Subscripts

1	upper wall
2	lower wall
w	wall

transport characteristics of the system. We would like to mention here that, a few such studies are also reported in the literature for analyzing the thermo-hydrodynamics of both the Newtonian and non-Newtonian fluids taking the aforementioned thermal boundary conditions into account in the analysis [37–40]. Moreover, in analyzing the thermo-fluidic transport process under such boundary conditions, while a few studies are devoted only to a simple pressure driven flow [37,38]; the combined effects of the movement of either of the boundaries and the applied pressure gradient have been considered in some cases as well [39–41]. In all these studies, different parameters, including the effect of viscous dissipation and those determining the heat transfer characteristics have been established and their influential role on the thermal transport characteristics of the system has been discussed in a comprehensive way. Although the literature is rich in analyzing the convective heat transfer characteristics under asymmetric thermal boundary condition, such investigation for viscoelastic fluid is yet to be remain conducted.

Therefore, in this work, we carry out a thorough analysis on the microscale thermo-fluidic transport of a viscoelastic fluid in a parallel flat plate microchannel subjected to asymmetric wall heat flux, by including the interfacial hydrodynamic slip and viscous dissipation effect. This investigation, carried out using an analytical framework, primarily explores the effects of thermal asymmetricity and viscous dissipation on the thermal transport of heat and its consequences on the irreversibility generation characteristics of the system. The rest of the paper is organized as follows: we formulate the problem in Section 2 and provide the explicit expressions for velocity and temperature distributions and also for the Nusselt number. In Section 3, the global entropy generation rate is computed and the interactive effects of the flow parameters on the temperature field, Nusselt number, and irreversibility generation rate are shown graphically and discussed in details. Finally, we draw the conclusions in Section 4 obtained from this analysis.

2. Statement of the problem and mathematical formulation

2.1. Physical considerations

The physical model under consideration is schematically shown in Fig. 1. We consider a parallel plate microchannel formed between two infinite flat plates of length L, width b and separated by a gap of $2H$ with $2H \ll L, b$. The coordinate axes x and y run along the stream wise and transverse directions of the channel respectively. An incompressible viscoelastic fluid flows through the microchannel under an externally applied pressure gradient dp/dx. Both walls of the channel are under the impression of a uniform but different heat flux q_1 and q_2 at the upper and lower walls respectively. This consideration ensures an asymmetric heat trans-

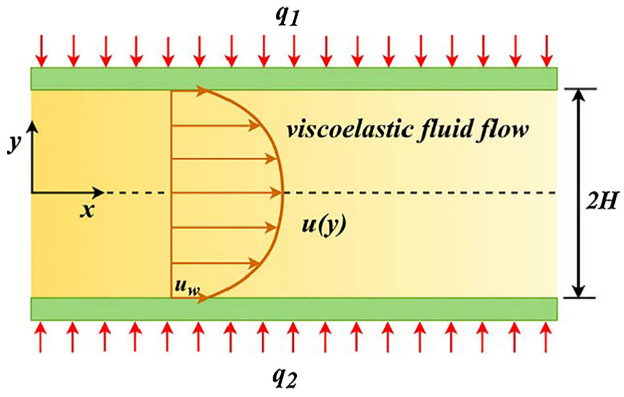

Fig. 1. Schematic depicting the physical geometry and the imposed boundary conditions. The externally applied pressure gradient drives a viscoelastic fluid through a parallel plate microchannel in the axial (x) direction of the channel.