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Numerical solution of multi-dimensional transient nonlinear heat conduction problems with heat sources by an extended element differential method

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ABSTRACT

In this paper, the element differential method is extended to solve a transient nonlinear heat conduction problem with a heat source and temperature-dependent thermophysical properties for the first time. The transient term is discretized by employing a finite difference scheme. An iterative methodology is developed to deal with the nonlinearity caused by temperature-dependent thermophysical properties. Examples of two-dimensional (2D) and three-dimensional (3D) problems are given to validate the present method for solving multi-dimensional transient nonlinear heat conduction problems. The results show that the present EDM provides a promising way that is effective and with high accuracy for solving multi-dimensional transient nonlinear heat conduction problems.

1. Introduction

Solving multi-dimensional transient nonlinear heat conduction problems is of extreme importance in many engineering applications [1–5], in which multi-dimensionality, transient term and temperature-dependent thermophysical properties are all involved. For solutions of heat conduction problems, there are mainly three ways: analytical, numerical, and experimental. The analytical solutions are limited to very simple problems, while experiments are always expensive or difficult to be carried out. Therefore, numerical approaches are dominant, and various techniques have been proposed, such as the finite difference method (FDM) [6,7], finite volume method (FVM) [8,9], finite element method (FEM) [10,11], meshless method (MLM) [12,13], boundary element method (BEM) [14–16] and some other innovative techniques [17–19].

Each method has its advantage and disadvantage, which has been reviewed in detail in Ref. [20]. As for transient nonlinear heat conduction problems, FDM is more suitable to solve problem with simple geometric configurations, considering accuracy, stability and efficiency. FEM is most powerful for solving complicated problems with complex geometries, and several commercial software is available, such as ANSYS, ABAQUS, NASTRAN and so on. However, the FEM in the commercial software is difficult to be applied in inverse analysis [21–23] by using the complex variable differentiation method for determining sensitivity coefficients. BEM would require further treatment of residual domain integrals caused by the transient term and the nonlinearity.

Although heat conduction problems have been extensively investigated, few references, associated with new techniques, have simultaneously dealt with heat source, multi-dimensionality, transient term, and temperature-dependent thermophysical properties. This paper presents a general approach and attempts to solve multi-dimensional transient nonlinear heat conduction problems with heat sources.

Recently, Gao and coworkers have proposed a new approach [20,24,25], element differential method (EDM), for solving second-order differential equations. This approach is based on the use of isoparametric elements as used in the standard FEM. A set of explicit formulations to compute the first and second order spatial derivatives were derived for two-dimensional and three-dimensional problems. No any mathematical principles or integrations are required. EDM is a strong-form technique, and the most important feature of the proposed method is that the derived spatial derivatives can be directly substituted into the governing equation and boundary conditions to form the final system of algebraic equations. Therefore, the EDM is very easy to code and program in dealing with engineering problems with complicated governing

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Nomenclature

A	coefficients matrix
b	vector
c	mass specific heat, J/(kg °C)
h	heat convective coefficient, W/(m^2 °C)
J	Jacobian matrix
L	1D shape function
i, j, k, l, n	subscripts
m	the number of interpolation points
N	order of equations
N_α	shape function
Q	heat source, W/m^3
q	heat flux, W/m^2
R	residual
T	temperature, °C
t	time, s
w	relaxation factor
$\boldsymbol{x}$	vector of unknowns
x_i	*the* i^{th} coordinate, m
x	x coordinate, m
y	y coordinate, m
z	z coordinate, m

Greek symbols

Γ	the boundary
Δ	change of variable
δ	Kronecker delta symbol
ρ	density, kg/m^3
ζ	intrinsic coordinate, m
η	intrinsic coordinate, m
λ	thermal conductivity, W/(m °C)
ξ	intrinsic coordinate, m

Subscripts

∞	surrounding

Superscripts

0	previous
k	kth iteration
m	mth

equations and boundary conditions, and the potential of the EDM in engineering applications has been partly validated in Ref. [20].

In the previous work, only the steady-state was considered. In this paper, the element differential method is extended to solve multi-dimensional transient nonlinear heat conduction problems with heat sources for the first time.

2. Transient nonlinear heat conduction problem

The governing equation of the transient nonlinear heat conduction problem with temperature-dependent thermophysical properties and a heat source can be expressed as

$$\frac{\partial}{\partial x_i}\left(\lambda_{ij}[T(x,t)]\frac{\partial T(x,t)}{\partial x_j}\right)+Q(x)=\rho[T(x.t)]c[T(x.t)]\frac{\partial T(x,t)}{\partial t}(x \in \Omega) \tag{1}$$

The boundary conditions are

$$T(x,t)=\overline{T} \qquad x\in\Gamma_1 \tag{2}$$

$$q(x,t)=-\lambda_{ij}[T(x,t)]\frac{\partial T(x,t)}{\partial x_j}n_i=\bar{q} \qquad x\in\Gamma_2 \tag{3}$$

$$q(x,t)=h[T(x,t)-T_\infty] \qquad x\in\Gamma_3 \tag{4}$$

where $\Gamma_1\cup\Gamma_2\cup\Gamma_3=\Gamma$, n is the outward normal to the boundary Γ, h is the heat convective coefficient; $\overline{T}$, $\bar{q}$ and T_∞ are the prescribed temperature, heat flux and surrounding temperature on the boundary, respectively. x_i is the i-th component of the spatial coordinates at point x, λ is thermal conductivity, ρ is density, c is mass specific heat, T is temperature, t represents time, and Q represents the heat source. The repeated subscripts i and j represent the summation through its range which is 2 for 2D and 3 for 3D problems.

3. Element differential method for solving transient nonlinear heat conduction problems

3.1. Shape functions

The heat conduction problem consists of first and second order partial derivatives of temperatures with respect to spatial coordinates. The derivatives are derived by using isoparametric elements. The key mathematical quantities characterizing the isoparametric elements are the shape functions. The shape functions for 1D isoparametric elements can be determined by the Lagrange interpolation formulation [20]:

$$L_I^m(\xi)=\prod_{i=1,i\neq I}^{m}\frac{\xi-\xi_i}{\xi_I-\xi_i} \qquad (I=1\sim m,-1\leqslant\xi\leqslant1) \tag{5}$$

where m is the number of interpolation points, ξ is the isoparametric coordinate. The shape functions for 2D and 3D problems can be formed based on the 1D shape functions, which can be expressed as follows.

$$N_\alpha(\xi,\eta)=L_I^m(\xi)L_J^n(\eta) \tag{6}$$

for 2D elements, and

$$N_\alpha(\xi,\eta,\zeta)=L_I^m(\xi)L_J^n(\eta)L_K^p(\zeta) \tag{7}$$

for 3D elements.

In Eqs. (5)–(7), the superscripts m, n, and p are the numbers of the interpolation points along ξ, η and ς directions, respectively, and the subscript α is determined by the permutation of the subscripts I, J, and K (for 3D case) [20,24].

3.2. Derivatives of elemental shape functions with respect to global coordinates

Any physical quantity varying over an isoparametric element can be expressed in terms of their nodal values of the element:

$$x_i=N_\alpha x_i^\alpha \tag{8}$$

$$T=N_\alpha T^\alpha \tag{9}$$

where x_i^α and T^α are the values of coordinates and temperature at node α, and the repeated index α represents the summation over all nodes. Based on Eqs. (8) and (9), one can obtain the first and second order derivatives:

$$\frac{\partial T}{\partial x_i}=\frac{\partial N_\alpha}{\partial x_i}T^\alpha \tag{10}$$