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Optimization analysis of fractal tree-like microchannel network for electroviscous flow to realize minimum hydraulic resistance

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ABSTRACT

As a significant principle to guide the optimization design of fractal tree-like network, Murray's law is obtained under the assumption of neglecting surface charge at the solid-liquid interface. However, surface charge should be considered due to the surface charge-induced electroviscous effect on the fluid flow over the micro/nanoscale. The present work carries out the optimization analysis of fractal tree-like microchannel network for electroviscous flow to realize minimum hydraulic resistance under the constraint of constant channel volume by considering the surface charge. It is found that the surface charge significantly modifies the conventional Murray's law. Both zeta potential to manifest surface charge and microchannel radius make non-monotonic effects on the optimal radius ratio between the daughter channel and the parent channel for the electroviscous flow in the fractal tree-like microchannel network to achieve a minimum hydraulic resistance. These non–monotonic effects are related to the apparent electro-viscosity in the microchannel with different channel radius and different zeta potential. Additionally, the optimal radius ratio is found to be linearly dependent on both the ratio of apparent electro-viscosities of electroviscous flow in the microchannels at the two successive branching levels and the length ratio of the microchannels at two successive branching levels.

1. Introduction

Inspired by tree-like topological structures (such as leaf veins of plants or tree branches) abundant in nature [1–3], tree-like branched networks have been paid comprehensive scientific attentions and widely used in various fields [4–10]. For example, the microchannel heat sink with fractal tree-like channel layout is found to have a better convective heat transfer performance and temperature uniformity compared to the conventional parallel microchannel heat sink [6–8].

In order to achieve optimal heat and mass transfer performances, the dimensional parameters and structural parameters of the tree-like branched network should be explicitly designed. Murray's law is a well-accepted optimization principle to guide the optimal design of tree-like branched networks for better heat and mass transfer performances [10–15]. For a self-similar one-to-multi branched structure with one parent branch and N daughter branches, the Murray's law can be generally expressed as following.

$$\beta_m = N^{\Delta} \tag{1}$$

where β_m is optimal radius ratio between the daughter branch and the parent branch, and Δ is a constant relating to different applications. For example, Δ is $-1/3$ for laminar flow and $-7/3$ for turbulent flow to reach a minimum hydraulic resistance, and $-1/2$ for the heat conduction to reach a maximum thermal conductivity [10–15].

Although Murray's law can be used to guide the optimal design of a tree-like channel network for fluid flow to reach a minimum hydraulic resistance in a simple manner of Eq. (1), it was obtained under the assumption of neglecting solid-liquid interfacial properties. For the macrochannel network, the neglecting of interfacial properties may be reasonable, however, for the microchannel network, it is believed that some interfacial properties play significant roles on the microfluidic flow and should be considered [16–21]. Among many interfacial properties, surface charge at solid-liquid interfaces is the one should be considered during the analysis of the fluid flow in a single microchannel or a complicated microchannel network.

When a given solid surface is brought into contact with an electrolyte solution, the solid-liquid interface can be spontaneously charged due to different mechanisms, and the surface charge density varies with different solid-liquid interface [22–24]. Then, the surface charge causes redistribution of ions in the electrolyte solution and results in the formation of a so-called electric double layer

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(EDL) adjacent to the solid-liquid interface. When the electrolyte solution in a microchannel is driven by an external pressure to flow, the EDL within the liquid will exert an additional electrical body force in the opposite direction of the driven pressure on the fluid flow and reduce the flow velocity. This effect of surface charge-induced EDL on the pressure-driven flow in the microchannel is called electroviscous effect [24]. The electroviscous flow both in a single microchannel with different cross-sectional shape and in a complicated microchannel networks have been studied [17–20,24–30]. However, most of the previous studies focused on the fluidic behavior (including pressure drop, velocity, electrical potential, electric current, etc.) of the electroviscous flow in a microchannel networks, there is less study on the optimization analysis of the microchannel network for the electroviscous flow to reach a minimum hydraulic resistance.

To solve this problem, the present work carries out the optimization analysis of the fractal tree-like microchannel network for the electroviscous flow to reach a minimum hydraulic resistance by considering the surface charge at the solid-liquid interface. Furthermore, effects of zeta potential manifesting surface charge at the solid-liquid interface, the branching number and the dimensional parameters including the length ratio of microchannels at two successive branching levels and the radius of microchannel at the initial branching level on the optimal tree-like microchannel network to reach a minimum hydraulic resistance are studied and analyzed.

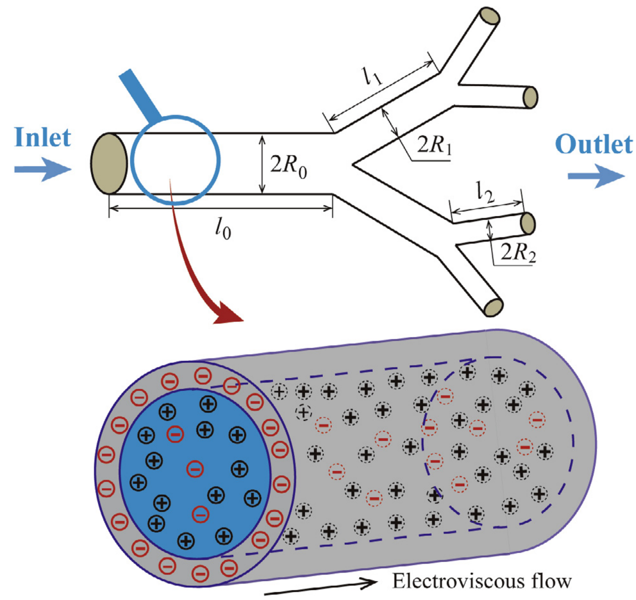

Fig. 1. A self-similar fractal tree-like microchannel network with $N = 2$ and $m = 2$ and simplified schematic of electroviscous flow in any microchannel induced by surface charge.

2. Generation of a self-similar fractal tree-like microchannel network

In present work, fractal tree-like microchannel network with self-similarity is used and all the microchannels are assumed to be in cylindrical shape. Being similar to the literatures [10,14], the self-similar fractal tree-like microchannel network is generated in the following manner. (1) Give a single microchannel at the 0th branching level with the fixed radius R_0 and length L_0. (2) From the 0th branching level, every microchannel is divided into N microchannel with the same radius and length at the next branching level. (3) The radius and length of every newly-generated microchannel satisfy the following scaling law.

$$\begin{cases} \gamma = l_{k+1}/l_k \\ \beta = R_{k+1}/R_k \end{cases} \quad (k = 0, 1, 2 \ldots\ldots, m) \tag{2}$$

where γ is the ratio between the length l_{k+1} of the microchannel at the $(k+1)$th level and the length l_k of the microchannel at the kth level, β is the ratio between the radius R_{k+1} of microchannel at the $(k+1)$th level and the radius R_k of the microchannel at the kth level, and m is the maximum branching level of the fractal tree-like network. Based on the above generation law, a typical self-similar fractal tree-like microchannel network with $N = 2$ and $m = 2$ is shown in Fig. 1. The simplified schematic of electroviscous flow induced by surface charge at the solid-liquid interface is also given in Fig. 1.

Furthermore, considering the radius and length of the microchannel at the 0th level is known, the radius and length of the microchannel at any branching level can be expressed as following.

$$\begin{cases} l_k = l_0 \gamma^k \\ R_k = R_0 \beta^k \end{cases} \tag{3}$$

3. Electroviscous flow in fractal tree-like microchannel network

To analyze the electroviscous flow in a fractal tree-like microchannel network, the following assumptions are made. (1) The electroviscous flow in every single microchannel is incompressible, laminar and fully-developed Newtonian flow; (2) although the junctions inevitably increase the total hydraulic resistance, here, the hydrodynamic development lengths is assumed to be very small compared to the length of the straight channels, then, effect of junctions on the hydraulic resistance can be neglected. This assumption is similar to previous studies [4–6]; (3) the surface charge at any solid-liquid interface keeps constant and uniform. Based on the above assumptions, the hydraulic resistance R_{Hk} of electroviscous flow in a single microchannel at kth branching level can be expressed as [25–27,31],

$$R_{Hk} = \frac{8\mu_{ek} l_k}{\pi R_k^4} \tag{4}$$

where μ_{ek} is the apparent electro-viscosity of the electroviscous flow in a single microchannel at kth branching level. The apparent electro-viscosity can be obtained from previous studies [25–27]. In present work, the apparent electro-viscosity is expressed as following [26,27].

$$\mu_{ek} = \mu \left[1 - \frac{8\beta^* (e\zeta/k_b T)^2 (1-G) F}{(\kappa R_k)^2} \right]^{-1} \tag{5}$$

where μ is the dynamic viscosity of the liquid, ζ is zeta potential to manifest the surface charge, $\kappa = (8\pi n_0 e^2/\varepsilon k_b T)^{1/2}$ is the Debye-Hückel parameter [26,27], n_0 is the liquid bulk ionic concentration, e is the elementary charge, ε is the liquid dielectric constant, k_b is the Boltzmann constant, T is the absolute temperature of the liquid, $\beta^* = (\varepsilon^2 k_b{}^2 T^2 \kappa^2 / 16\pi^2 \mu\lambda e^2)$ is a dimensionless parameter related to the liquid properties, λ is the liquid electrical conductivity, and G and F are functions of R_k and ζ, which can be found from previous reported studies [26,27].

Obviously, the apparent electro-viscosity is related to channel radius, zeta potential, electrical potential distribution within the EDL and properties of liquid, etc. The apparent electro-viscosity shows a non-monotonous variation with both the channel radius and the zeta potential. The electro-viscosity increases and then decreases with the increasing channel radius and the increasing zeta potential [17–19,25–27]. Thus, the electroviscous flow in a